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ABSTRACT

The Sequential Mathematics Sequence provides an integrated course of study for grades 9-11 in New York City. This document presents lesson plans for part two of the first-year course, designed for ninth grade. The curriculum interweaves algebra, geometry, logic, probability, statistics, and trigonometry, with the emphasis in the 60 lessons in this document on geometry and algebra. Each lesson begins with a clearly stated aim presented in question form. Performance objectives, new vocabulary, and a motivational activity follow. A full development is then presented, with suggested pivotal questions. Applications, a summary, and sample homework examples are also included. In each lesson opportunities are infused for helping students to develop critical thinking, problem-solving, and reasoning skills. (MNS)

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SEQUENTIAL MATHEMATICS

Course I

Term 2

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SEQUENTIAL MATHEMATICS

(formerly Integrated Mathematics)

Course I

Term 2

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Division of Curriculum and Instruction

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FOREWORD

The Sequential Mathematics Sequence provides New York City students with an integrated course of study for grades nine, ten, and eleven. In a departure from traditional courses of study, lessons interweave algebra, geometry, logic, probability, and statistics. The Sequential Mathematics Sequence extends the integrated approach that begins in kindergarten and continues through grade 8.

In the Sequential Mathematics Sequence, algebra, geometry, and trigonometry remain important components of the curriculum. In addition, other branches of mathematics --such as probability and statistics -- are introduced and emphasized as important tools for dealing with present and future technological developments.

Each lesson in Course I begins with a clearly stated aim presented in question form. Performance objectives follow; these provide a specific focus for the teacher. The new vocabulary for the lesson is presented. This is followed by a motivational activity. A full development is presented, with suggested pivotal questions. Applications, a summary, and sample homework examples are also included. In each lesson opportunities have been infused for helping students to develop critical thinking, problem-solving, and reasoning skills. These activities will enable students to become more skilled in higher levels of thinking.

The Sequential Mathematics Sequence has been designed to anticipate students' needs for mathematical skills in an increasingly technologically-oriented culture. We anticipate that the Sequential Mathematics Sequence will serve the individual needs of your students and enable them to acquire the mathematics skills they need in the present and for the future.

Charlotte Frank

Charlotte Frank
Executive Director
Division of Curriculum and Instruction

ACKNOWLEDGMENTS

Sequential Mathematics, Course I, Term 2 was developed for teachers of Sequential Mathematics as a project of the Mathematics Unit of the Division of Curriculum and Instruction, Charlotte Frank, Executive Director.

Morris Freedman, Director of the Office of Curriculum Development and Support, provided overall supervision of this project.

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INTRODUCTION

Sequential Mathematics (Course I), Part 2 is the second publication from the Division of Curriculum and Instruction, completing a guide for teachers of this newly implemented New York State curriculum. It attempts to present the material covered in this course in a way that is most comprehensive and useful to the classroom teacher.

Traditionally, in New York State, mathematics in grades nine, ten, and eleven has consisted of algebra, geometry, and intermediate algebra/trigonometry. Students completing the three-year sequence have been exposed to these topics as relatively independent units. Moreover, students discontinuing their study of mathematics after grade eleven have never been exposed to such timely and practical topics as probability, statistics, and formal logical reasoning. Those terminating their high school mathematics program after grade ten have been given even fewer opportunities for exposure to these topics.

The three-year Sequential Mathematics sequence designed by New York State is a move in the direction of fusing these formerly separate topics and introducing those which have been absent from the course of study.

This curriculum guide is based upon the Scope and Sequence for Sequential Mathematics, Course I, Part 2. It consists of lesson plans, which should be considered suggestions rather than prescriptions. Each lesson stresses the development of mathematical concepts, with an emphasis on providing opportunities for the development of problem solving skills, reasoning skills and critical thinking. To help students develop these skills, each lesson contains suggested questions to encourage student investigation and thought. The problems presented in the lessons are designed to be relevant to students' experiences. There is an abundance of application and homework problems. These have been provided to help teachers adapt instruction to the individual differences within the classroom.

Though Course I is written with ninth grade students in mind, it is possible that some students may begin the material earlier or later, or complete it in less than or more than one year. It is expected that individual teachers will consider their students' abilities and interests in making judgments concerning time allotments, aspects of material to be stressed, or extra material to be presented.

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Lesson 1 - Algebra and Geometry

AIM: What is geometry?

PERFORMANCE OBJECTIVES: The student will be able to...

1. Describe briefly the branch of mathematics called geometry.
2. State the difference between defined and undefined terms.
3. Give three undefined terms in geometry.
4. Classify angles (types of angles, parts of an angle).
5. Classify triangles (types of triangles, parts of a triangle).

VOCABULARY: point, line, plane, undefined term, line segment, ray, measure of an angle, vertex, right angle and triangle, acute angle and triangle, obtuse angle and triangle, scalene, isosceles, equilateral

MOTIVATION:

Ask: What is geometry? Elicit many student opinions.

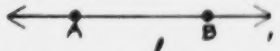
DEVELOPMENT:

1. List various points presented by the students, then elicit and write a brief description of geometry. Include the study of lines, angles, figures, circles and measurement. Also indicate that geometry in high school differs from previously learned geometry in that now students will formerly justify the truth of statements and will derive formulas to be used. This study of geometry will use many principles of logic.
2. a) Elicit a simple definition with a question such as, "What is a chair?" Keep asking for the definition of the words that the students use to indicate that there is a need for a starting place. These will be <u>undefined terms</u>. Undefined terms are terms we explain, show, understand, but don't define. The

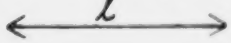
three undefined terms that are the foundation of all definitions to follow are point, line and plane.

b) Show a point, ., and name it with a capital letter, .P, point P. Ask, "What does a point do?" (Indicates location.)

c) Describe a line as a set of points that goes on indefinitely in two directions. Draw a line, and name it in two ways.



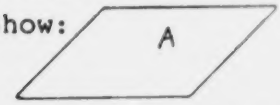
, $\overleftrightarrow{AB}$, line AB,

or 

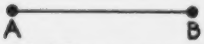
line l.

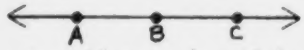
A line has no thickness or depth.

d) Describe a plane. It is represented by a flat surface. Ask for examples of planes in the room (desk, wall, floor, ceiling, etc.).

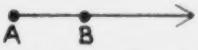
Show: 

, plane A.

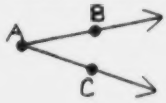
3. 

Draw a line segment. Ask, "What is the difference between a line and this figure?" Elicit that a line goes on indefinitely while this has two ends. Define this as a line segment. Definition: A line segment is part of a line with two endpoints. Name segments $\overline{AB}$ or $\overline{BA}$. Ask, "How many lines are in this figure?" 

(1) Ask, "How many line segments are there in this picture? (3) Name them." ($\overline{AB}$, $\overline{BC}$, $\overline{AC}$)

4. What is this? 

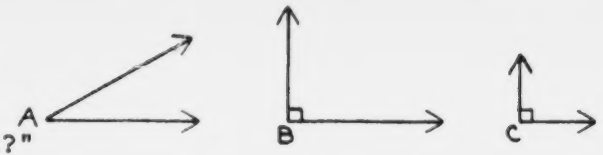
(a ray) Define a ray as part of a line with one endpoint. $\overrightarrow{AB}$ is ray AB. Illustrate that $\overrightarrow{AB}$ and $\overrightarrow{BA}$ name different figures.

5. Draw this figure 

and ask the students to describe it.

(Two rays, $\overrightarrow{AB}$ and $\overrightarrow{AC}$, with a common endpoint.) What do we call this? (an angle) Define: An angle is the union of two rays with a common endpoint. The endpoint is called the vertex of the angle. The rays are called the sides of the angle. Name the angle ∡BAC or ∡CAB or ∡A.

6. Draw these angles and ask, "Which of these angles is the smallest?"



Elicit that the sides of any angle are rays. The length of the portion of the rays shown in this diagram has no bearing on the size of the angle. The measure of an angle reflects the amount of <u>rotation</u> between the sides. ∡B and ∡C have equal measures (90° each). ∡A appears to contain less than 90°.

7. Angles may be classified according to their measures. Define:

   <u>acute angle</u>: an angle whose measure is more than 0° and less than 90°.

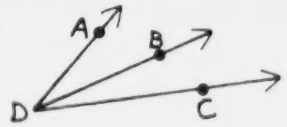
   <u>right angle</u>: an angle whose measure is 90°.

   <u>obtuse angle</u>: an angle whose measure is more than 90° and less than 180°.

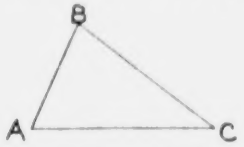
   <u>straight angle</u>: an angle whose measure is 180°.

   Note: Two <u>lines, segments or rays</u> which meet to form a right angle are said to be <u>perpendicular</u> (⊥).

8. Ask, "How many angles are in this diagram?" (3) Name them (∡ADB, ∡BDC, ∡ADC). What is the relationship between their measures? (m∡ADB + m∡BDC = m∡ADC.)

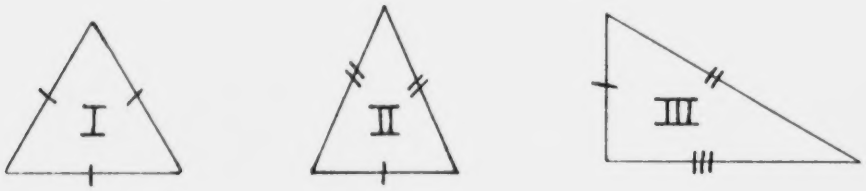


9. Draw a triangle.



Elicit that this is a triangle and the definition: A triangle is a figure with three sides, three angles and three vertices. Name the sides of the given triangle, $\overline{AB}$, $\overline{BC}$, $\overline{AC}$. Name the triangle (△ABC). Name the angles (∡A or ∡BAC, ∡B or ∡ABC, ∡C or ∡ACB).

10.

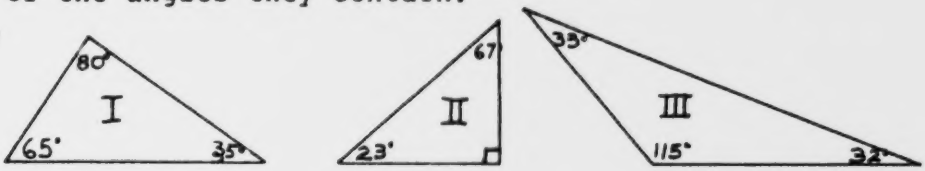


Look at the sides of these triangles. What seems to be true about △I? (The lengths of all three of its sides

are equal.) State that this is an <u>equilateral</u> triangle and write the definition. Similarly, elicit that △II is isosceles and write its definition (that two sides are equal). Ask, "Is △I also isosceles?" (yes) Elicit that △III has sides of three different lengths. It is called a scalene triangle.

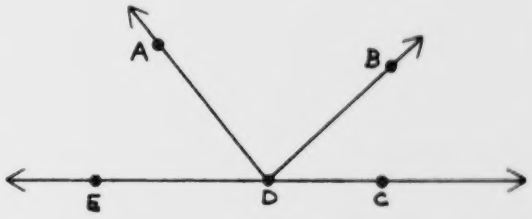
11. State: Triangles may also be classified according to the size of the angles they contain.

Draw:



Ask: What is notable about the angles in these triangles? Elicit: I has <u>all</u> acute angles, it is called an acute triangle. II has <u>a</u> right angle. What is it called? (a right triangle) III has <u>an</u> obtuse angle, therefore it is an obtuse triangle.

APPLICATIONS:



In this figure,

1. Name five points.
2. Name a line and five line segments.
3. Name four rays.
4. Name three angles containing $\overrightarrow{DA}$.
5. Name three angles that don't contain $\overrightarrow{DA}$.
6. Name the kind of angle:

7. Triangle ABC has angles that measure 50°, 60° and 70°. What kind of triangle is △ABC?
8. Name the type of triangles whose angle measures are listed below.
   a) 40°,50°,90° b) 20°,50°,110° c) 80°,80°,20°
9. Name the types of triangles whose sides measure as follows.
   a) 12cm,18cm,20cm b) 30cm,50cm,30cm c) 5cm,5cm,5cm

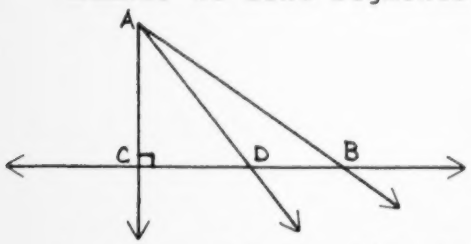
SUMMARY:

1. What are some undefined terms in geometry?
2. Can a triangle be both isosceles and equilateral? Explain.
3. What are the types of angles we have learned so far?
4. Draw a triangle which is obtuse and scalene.
5. Draw a triangle which is obtuse and isosceles.

HOMEWORK:

1. Your dictionary defines "point". Why do we classify "point" as undefined in geometry?
2. Show four points, A, B, C, D
   a) all in a line.
   b) at most, 3 in a line.
   c) at most, 2 in a line.
   d) How many lines are determined when at most 2 of the 4 points are in a line? (Draw them.)
3. Show 5 points, A, B, C, D, E, with at most 2 points in one line. How many line segments can you draw connecting 2 of these 5 points? Fill in the table as far as you can and try to discover a pattern.

Number of Points	1	2	3	4	5	6
Number of Line Segments						

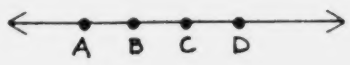
4. 

   a) Name 3 triangles and classify them according to angles they contain.
   b) Name one angle for each type learned so far.
   c) Name all the line segments.
   d) Name 6 rays.
   e) Name one line.
5. How many equilateral triangles can you make with 6 toothpicks, each a side of at least one triangle?
6. Can a triangle have more than one
   a) right angle? b) obtuse angle?

7. Can an isosceles triangle also be a right triangle? Explain.

8. Can a right triangle be scalene? Equilateral? Explain.

9. How many line segments are in the figure:

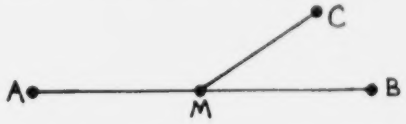


Fill in the table and try to find a pattern.

No. of points in the line	1	2	3	4	5	6
No. of line segments						

10. Draw a triangle that is both
    a) obtuse and isosceles.
    b) obtuse and scalene.
    c) acute and scalene.

11. How many degrees are there in $\measuredangle AMC + \measuredangle CMB$? (AB is a straight line.)



12. What is the difference between a straight line and a straight angle?

13. In isosceles triangle ABC, $\overline{AB}$ and $\overline{BC}$ have equal lengths. If AB = 3x + 2, AC = 5x - 10, BC = 2x + 12, find AB, AC, BC.

Lesson 2 - Geometry

AIM: What is a circle?

PERFORMANCE OBJECTIVES: The student will be able to...

1. Define circle.
2. Define, identify and label:
   a) circle
   b) radius
   c) diameter
   d) central angle
   e) chord
   f) inscribed angle
3. Solve numerical and algebraic problems involving radii and diameters of a circle.

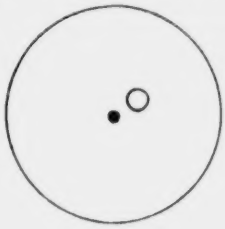
VOCABULARY: circle, center, radius, diameter, central angle, chord, inscribed angle

MOTIVATION:

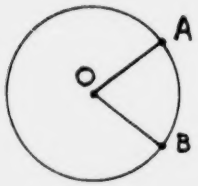
A treasure is buried 10 feet from a tree; where would you dig to find the treasure?
The teacher should draw a tree on the chalkboard and ask students to place a dot where they would dig.

DEVELOPMENT:

1. After many dots are drawn, ask, "What figure do we seem to have?" Elicit that the points form a circle and elicit and write the definition of a circle on the chalkboard. A <u>circle</u> is the set of all points that lie on a plane so that each is the same distance from a given point.
   Ask: What is the name of the "given point" in our definition? (center)
   Inform the students that we usually identify a circle by naming its center. This is circle O. (⊙ O)



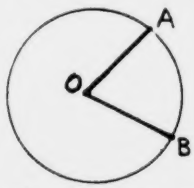
2. Draw $\overline{OA}$ from the center to the circle.
Define radius: a segment whose endpoints are the center and a point on the circle.
Draw $\overline{OB}$.



Ask: What is true about $\overline{OA}$ and $\overline{OB}$? Elicit that they are equal in length. Why?

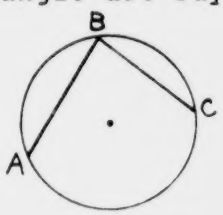
3. Draw a chord $\overline{XY}$. Define a chord: a line segment that has both of its endpoints on a circle.

4. Extend $\overline{AO}$ to meet $\odot O$ in C. $\overline{AC}$ is called a diameter.
Define <u>diameter</u>: a chord which passes through the center. What is the relationship between a diameter and a radius of a circle? (d = 2r). Ask, "Is a diameter always a chord? (Yes.) Is a chord always a diameter?" (No.) Explain.



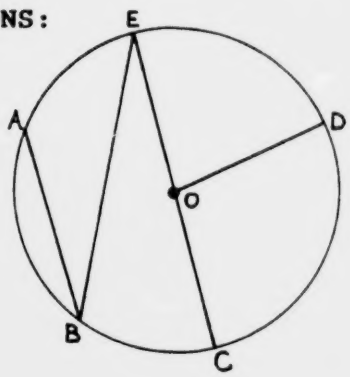
5. Consider angle AOB.
Where is its vertex?
This is called a <u>central angle</u>.
Elicit the definition: A central angle is an angle whose vertex is the center of a circle and whose sides contain radii. (Note that the sides of an angle are rays.)

6. Is $\measuredangle ABC$ a central angle?
Where is its vertex?
This is called an <u>inscribed angle</u>.
Elicit the definition:
An inscribed angle is an angle whose vertex lies on a circle and whose sides contain chords of a circle.



APPLICATIONS:

1.



a) name the circle
b) name 3 radii
c) name 3 chords
d) name a diameter
e) name 3 central angles
f) name 2 inscribed angles

2. Find the length of a diameter of the circle if the radius is:

   a) 12 cm b) 6 cm c) 5.5 cm d) $2\frac{1}{2}$ cm e) x cm

3. Find the radius of a circle whose diameter is:

   a) 10 cm b) 18 cm c) 15 cm d) 9.4 cm e) x cm

4. If radius $\overline{OA} = 3x + 10$ and radius $\overline{OB} = 5x - 8$ find

   a) the radius of the circle;

   b) the diameter.

5. If diameter $\overline{AB} = 5y + 8$ and radius $\overline{OB} = 3y - 2$, find OA.

SUMMARY:

1. What is a chord of a circle? Can a chord be shorter than a radius? ...longer than a radius? ...longer than a diameter?

2. If a triangle has two sides which are radii and one side which is a chord, what type of triangle is it?

3. Describe the difference between a central angle and an inscribed angle.

HOMEWORK:

1. How many chords connect 3 points on a circle? 4 points? Fill in the table and try to discover a pattern:

No. of points on circle	1	2	3	4	5	6
No. of possible chords						

2. Using a compass, draw a circle. Label its center O. Let $\overline{AB}$ be a diameter of $\odot$ ◯ . Let P, Q, R be any points other than A, B on $\odot$ ◯. With your protractor find the number of degrees in (after drawing the appropriate chords)

   a) ∡APB b) ∡AQB c) ∡ARB

   What kind of angle is each? What generalization can you make from your three measurements?

3. Let $\overline{AB}$ be a chord of circle O of length equal to the radius. Let P, Q, R be points on the larger arc AB. With your protractor, find the number of degrees in

(after drawing the appropriate chords)

a) ∡APB b) ∡AQB c) ∡ARB

What generalization might you make based on your three measurements?

4. In circle O find whether the center, O, falls inside, on or outside △ABC when

   a) △ABC is acute. b) △ABC is obtuse.

   c) △ABC is right.

   Draw each triangle to find out. (Points A, B and C lie on the circle.)

5. If a circle has a radius of 5", what is the length of its largest chord? ...of its shortest chord?

6. Radii $\overline{OA}$ and $\overline{OB}$ are drawn in circle O. Chord $\overline{AB}$ is also drawn to form △OAB. Answer TRUE or FALSE and try to justify your choice.

   a) △AOB is isosceles.

   b) △AOB may be a right triangle.

   c) △AOB may be a scalene triangle.

   d) △AOB may be an acute triangle.

   e) △AOB may be an obtuse triangle.

   f) ∡A may be obtuse.

7. If all the points placed on a circle are connected by chords, what is the nmumber of triangles that can be formed with vertices on the circle if the number of points is 3? ...is 4? Fill in the table and try to discover a generalization.

Number of Points	1	2	3	4	5	6
Number of Triangles						

8. Starting with any point on circle O, A, find a point B so that chord $\overline{AB}$ equals the length of a radius. Now find a point C so that chord $\overline{BC}$ equals the length of a radius. How many such chords can you find? Explain.

Lesson 3 - Geometry

AIM: How do we find the measure of an inscribed angle?

PERFORMANCE OBJECTIVES: The student will be able to...

1. Define the relationship between an inscribed angle and its intercepted arc.
2. Solve numerical problems involving the measures of:
   a) radius and diameter
   b) central angle
   c) inscribed angle

VOCABULARY: intercepted arc

MOTIVATION:

In each case, find the measure of angle AOB.

fig. 1

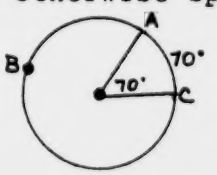
fig. 2

fig. 3

DEVELOPMENT:

1. Elicit that ∠AOB is a central angle. Since the circle is divided into equal parts, each part or ARC has the same measure. An arc is a part of a circle with two end points on the circle. The whole circle measures 360°. In figure 1, $m\overset{\frown}{AB}$ = 120°. In figure 2, $m\overset{\frown}{AB}$ = 90°. In figure 3, $m\overset{\frown}{AB}$ = 72°.

   Elicit and write on the board: A central angle is equal in measure to its intercepted arc. At this point define intercepted arc: An intercepted arc is an arc (part of a circle) cut off by a chord or an angle. (For brighter students: smaller arc = minor arc, larger arc = major arc; unless otherwise specified, the minor arc is intended.)



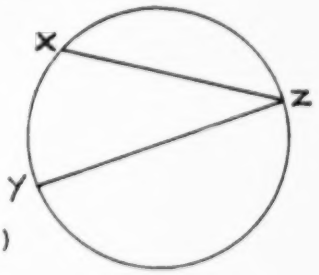
minor arc AC = 70°

major arc ABC = 360 - 70 = 290°

2. Have each student draw a circle with a central angle of 90°. Have each student draw an inscribed angle that intercepts the 90° arc. Measure this angle with a protractor (it is 45°).
   In the same figure, draw another inscribed angle which intercepts the same arc. What is its measure? Repeat - the result is 45°. Observe that all students obtained the same result, although the radii of circles they drew were of different lengths.
   What seems to be the relationship between the size of the inscribed angle and the size of the intercepted arc? What should we do to see if this is true for arcs other than 90°?

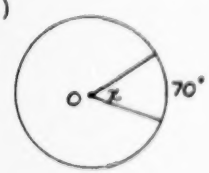
3. Divide the class into groups and have them repeat the experiment, starting with different central angles (and therefore different arcs). Suggested: 60°, 100°, 120°, 150°. Now have the students measure the inscribed angles. Put the data on the board and elicit: An inscribed angle is measured by one half of its intercepted arc.
   This conclusion is based on an induction from experimental data.

4. If arc XY measures 80°, what is the measure of inscribed angle XZY? (40°)
   If the measure of ∠XYZ equals 50°, what is the measure of $\overset{\frown}{XY}$? (100°)



APPLICATIONS:

1. Find the value of x in each circle.

a)


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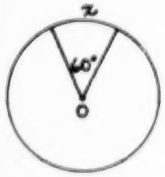

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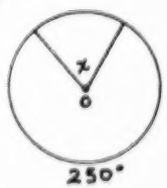

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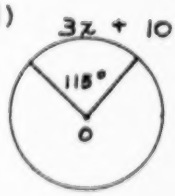
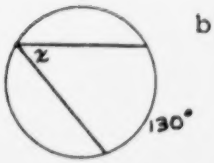
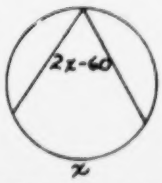
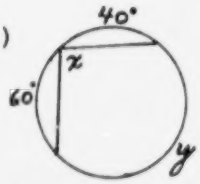


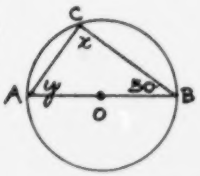
What kind of angles are these?

2. Find the measure of each angle or arc represented by x or y.

a) 

b) 

c) 

d) 

3. What is the measure of a central angle whose intercepted arc measures 110°?

4. What is the measure of an inscribed angle that intercepts an arc whose central angle equals 80°?

5. In $\odot\bigcirc$, inscribed $\measuredangle ABC$ measures 70°. What is the measure of $\overset{\frown}{AC}$?

6. In $\odot\bigcirc$, $\overline{AB}$ is a diameter. C is on $\odot\bigcirc$. Find x if the measure of $\measuredangle ACB$ is equal to $2x + 20$.

SUMMARY:

1. What is the relationship between an inscribed angle and its intercepted arc?

2. What is the relationship between two inscribed angles which intercept the same arc?

3. What is the relationship between an inscribed angle and a central angle which intercept the same arc?

HOMEWORK:

1. Three points, A, B, C, are equally spread around circle O and joined by chords to form $\triangle ABC$. How many degrees are there in

   a) $\overset{\frown}{AB}$
   b) $\overset{\frown}{AC}$
   c) $\overset{\frown}{BC}$
   d) $\measuredangle A$
   e) $\measuredangle B$
   f) $\measuredangle C$

2. Five points, A, B, C, D, E, are equally spaced around circle O and joined in order by chords to form polygon ABCDE. Chord $\overline{AC}$ is drawn. How many degrees are there in

   a) $\overset{\frown}{AB}$
   b) $\overset{\frown}{BC}$
   c) $\overset{\frown}{CD}$
   d) $\measuredangle ABC$
   e) $\measuredangle ACD$
   f) $\measuredangle ACB$
   g) $\measuredangle BAC$
   h) $\measuredangle D$
   i) $\overset{\frown}{AED}$

3. Three points, A, B, C, are placed around circle O so that $\overset{\frown}{AB}:\overset{\frown}{BC}:\overset{\frown}{CA} = 2:3:4$. Chords are then drawn to form $\triangle ABC$. How many degrees are there in

a) $\overset{\frown}{AB}$ d) $\measuredangle A$

b) $\overset{\frown}{BC}$ e) $\measuredangle B$

c) $\overset{\frown}{CA}$ f) $\measuredangle C$

g) What is the continued ratio for the measures $m\measuredangle A : m\measuredangle B : m\measuredangle C$?

4. Four points, A, B, C, D, are placed around circle O so that $\overset{\frown}{AB}:\overset{\frown}{BC}:\overset{\frown}{CD}:\overset{\frown}{DA} = 1:2:3:4$. Chords are then drawn in order to form polygon ABCD. Diagonals $\overline{AC}$ and $\overline{BD}$ are drawn. How many degrees are there in

a) $\overset{\frown}{AB}$ e) $\measuredangle ABC$ i) $\measuredangle ACD$

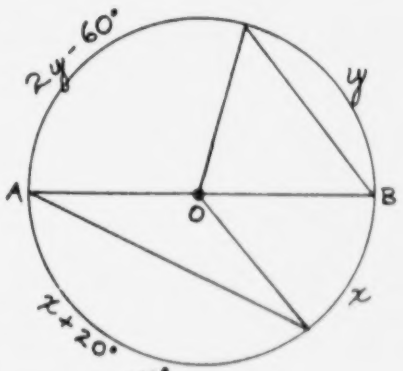
b) $\overset{\frown}{BC}$ f) $\measuredangle BCD$ j) $\measuredangle BDC$

c) $\overset{\frown}{CD}$ g) $\measuredangle CDE$ k) $\measuredangle BDE$

d) $\overset{\frown}{DE}$ h) $\measuredangle DEA$ l) an acute angle formed by the diagonals.

What seems to be true about the sum of the measures of a pair of opposite angles: $\measuredangle A + \measuredangle C$ or $\measuredangle B + \measuredangle D$? Try to show that your conjecture holds for every inscribed quadrilateral.

5. $\overline{AB}$ is a diameter for circle O. Using the information shown, find the number of degrees in



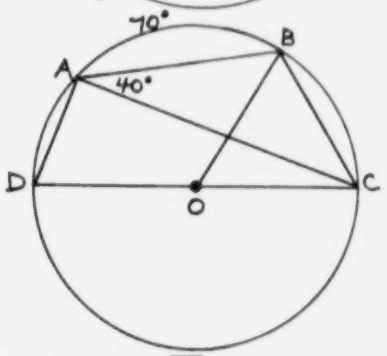
a) $\overset{\frown}{BC}$ e) $\measuredangle BOC$

b) $\overset{\frown}{CA}$ f) $\measuredangle COA$

c) $\overset{\frown}{BD}$ g) $\measuredangle ABD$

D) $\overset{\frown}{DA}$ h) $\measuredangle A$

6. In circle O radii $\overline{OB}$ and $\overline{OC}$ are shown. Chords $\overline{AB}$ and $\overline{BC}$ are drawn. $\overline{COD}$ is a diameter. Find the number of degrees in



a) $\overset{\frown}{BC}$ e) $\measuredangle BCA$

b) $\overset{\frown}{AD}$ f) $\measuredangle ACD$

c) $\measuredangle BOC$ g) $\measuredangle DAC$

d) $\measuredangle BOD$ h) $\measuredangle ABO$ (Hint: First draw $\overline{OA}$.)

Lesson 4 - Geometry

AIM: What are complementary and supplementary angles?

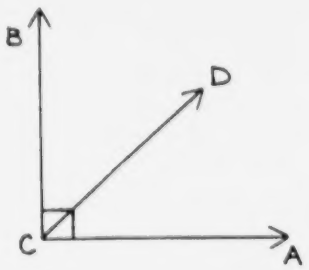
PERFORMANCE OBJECTIVES: The student will be able to...

1. Define
   a) complementary angles
   b) supplementary angles
2. Solve numerical and algebraic problems dealing with complementary and supplementary angles.

VOCABULARY: complementary angles, supplementary angles, adjacent angles

MOTIVATION:

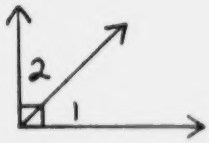
If m∠ACD = 70°, what is the measure of ∠BCD?



DEVELOPMENT:

1. Elicit that ∠BCA is a right angle and measures 90°. From this the sum of the measures of ∠ACD and ∠BCD = 90°. Therefore, m∠BCD = 90 - 70 = 20°.
   State that when two angles, such as ∠ACD and ∠BCD, have measures which add up to 90°, they are called complementary angles. Each may also be called the complement of the other. Elicit the definition: Two angles are called complementary angles if and only if the sum of their measures equals 90°.
   Show two angles with the correct sum which are not adjacent. Ask: "Are these angles complementary?" (They are; the definition says nothing about the orientation of the angles, but only requires that the measures add up to 90°.)
   Show three angles that measure 15, 25 and 50°. Ask, "Are these angles complementary angles?" (No, only two angles can be complementary.)

Ask, "Will angles 1 and 2 always be complementary in this diagram? (Yes) Why?"



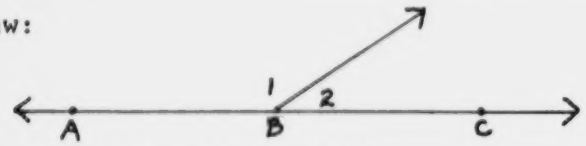
The term adjacent angles should be defined: Two angles with a common vertex and common side between them are called <u>adjacent</u> angles. Elicit: If two <u>adjacent</u> angles form a right angle, then they are complementary angles.

Ask, "If ∡A and ∡B are complementary and ∡A measures 37°, what is the measure of ∡B?

What is the measure of the complement of an angle of 42°?"

Continue with other similar simple examples.

2. Draw:



Ask, "What is m∡1 + m∡2?"

Elicit: ∡ABC is a straight angle that measures 180°, therefore ∡1 + ∡2 = 180°.

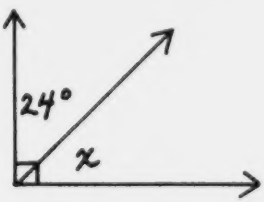
State that these are called supplementary angles and follow the same procedure as with complementary angles. That is:

a) Elicit the definition.
b) Show nonadjacent supplementary angles.
c) Show three angles totaling 180° and conclude that the term "supplementary" may not be applied.
d) Give numerical exercises to have students find one of a pair of supplementary angles.
e) Elicit: If two adjacent angles form a straight angle, then they are supplementary.

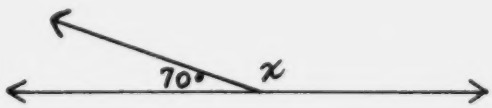
APPLICATIONS:

1. Find x in each case.

a)

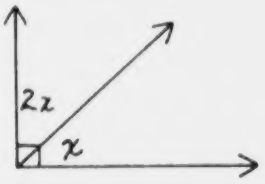


b)

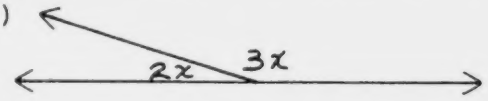


2. Find the complement and the supplement of each angle.
   a) $35°$ b) $19\frac{1}{2}°$ c) $x°$
3. If one of two complementary angles measures $80°$, what is the measure of the other?
4. If one of a pair of supplementary angles measures $80°$, what is the measure of the other?
5. Find x in each case, then find the measure of each angle.

a)



b)



6. Two complementary angles measure $(x + 15)°$ and $(3x - 5)°$. Find the measure for each angle.
   Note: For slower classes first ask to find x.
7. Two supplementary angles measure $(2y - 12)°$ and $(5y + 17)°$. Find the measure of each angle.
8. One angle is $20°$ more than its supplement. Find the measure of both angles.
9. If an angle is 9 less than twice its complement, find the measure of the larger angle.
10. If two supplementary angles are in the ratio of 5:4, find the measure of the larger angle.

SUMMARY:

1. What are complementary angles?
2. What are supplementary angles?
3. What angle has the same measure as its complement?
4. What is the supplement of the supplement of $15°$?
5. Which is larger, the complement of $32°$ or the supplement of $32°$? How much larger? Will this be true of all angles?

HOMEWORK:

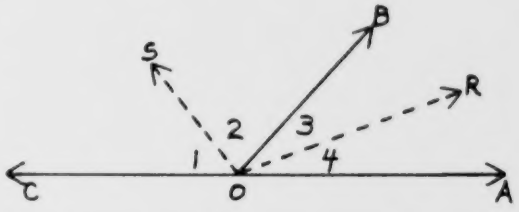
1. The ratio of two complementary angles is 2:3. What are their measures?

2. One complementary angle is 10° more than twice the other. How large is the smaller angle?

3. The ratio of two supplementary angles is 3:5. How much does each measure?

4. One supplementary angle is 30° less than three times the other. How large is the greater angle?

5. $\overleftrightarrow{AOC}$ is a straight line. $\overrightarrow{OR}$ bisects ∠AOB (cuts it in half). $\overrightarrow{OS}$ bisects ∠BOC.



   a) Show that $\overrightarrow{OS} \perp \overrightarrow{OR}$.
   b) Name 3 pairs of complementary angles.
   c) Name 3 pairs of supplementary angles.
   d) If m∠AOB = 50 , find the measures of:

1) ∠AOR	5) ∠ROS
2) ∠BOC	6) ∠ROC
3) ∠BOS	7) ∠BOR
4) ∠AOS	8) ∠COS

6. A quadrilateral is inscribed in a circle (all four of its vertices lie on the circle). Show that the opposite angles are supplementary.

7. If an angle is acute, what type of angle will its complement be? What type of angle will its supplement be?

8. What is the difference in the measures between the supplement and complement of 75°? ...of 25°? ...of n°?

9. a) When is an angle equal to its own complement?
   b) When is an angle twice its own complement?
   c) When is an angle equal to its own supplement?
   d) When is an angle equal to three times its supplement?

10. What is the sum of the measures of the supplement and complement of 75°? ...of 25°? ...of n°?

11. The supplement of an angle is 4 times its complement. How large is the angle?

12. The supplement of the complement of an angle is 100°. How large is this angle?

13. The ratio of the complement of an angle to its supplement is 2:5. How large is this angle?

14. TRUE or FALSE. Explain your answer.
    a) The supplement of an angle is always greater than its complement.
    b) The 3 angles of a triangle add to 180° so they must be supplementary.

Lesson 5 - Geometry

AIM: What are vertical angles?
What is a postulate?
What is a theorem?

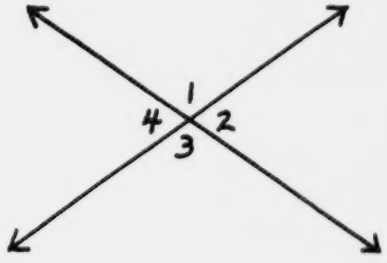
PERFORMANCE OBJECTIVES: The student will be able to...

1. Define vertical angles.
2. Informally and formally verify that vertical angles have the same measure.
3. Define congruent.
4. Solve numerical and algebraic problems dealing with vertical angles.
5. Define "postulate" and "theorem."

VOCABULARY: congruent, nonadjacent, vertical, postulate, theorem

MOTIVATION:

Two lines intersect so that m∡4 = 70°. Can the measures of any other angles be found?



DEVELOPMENT:

1. Elicit that ∡4 and ∡1 (also, ∡4 and ∡3)
   a) are pairs of adjacent angles and
   b) are pairs of supplementary angles.

   Therefore, m∡1 equals m∡3 = 110°. Similarly, ∡2 is supplementary to ∡1 (or ∡3), so its measure must be 70°.
   Ask, "How were all of these angles formed?" (by two intersecting straight lines)
   Ask, "What do all of these angles have in common?" (the same vertex)
   Observe that ∡1 and ∡3 are not adjacent or "nonadjacent" angles. Ask the students to name a second pair of nonadjacent angles in the figure (∡2 and ∡4).

Provide the definition: Vertical angles are two non-adjacent angles formed by two intersecting straight lines.

2. What is true about the measures of the vertical angles in this figure? (They have the same measure, they are equal in measure.) State that we call such angles congruent angles. Provide the definition of congruent angles: Two angles with the same measure are congruent ($\cong$). The teacher must emphasize that congruent refers to figures and measure refers to numbers. Angles are $\cong$, measures are =.

$\angle 1 \cong \angle 3$ $\quad\quad$ $m\angle 1 = m\angle 3$

$\angle 2 \cong \angle 4$ $\quad\quad$ $m\angle 2 = m\angle 4$

Ask: "Will this always be true about any pair of vertical angles?" Make a diagram for the general case and elicit:

$m\angle 1 + m\angle 2 = 180°$

$m\angle 2 + m\angle 3 = 180°$

$m\angle 1 + m\angle 2 = m\angle 2 + m\angle 3$

Taking away $m\angle 2$ from both sides leaves $m\angle 1 = m\angle 3$.

Therefore we conclude: Vertical angles are equal in measure; vertical angles are congruent.

Ask the class to recall the procedure used recently to establish the rule about the measure of inscribed angles (induction: the generalization was based on experimental data). Compare the procedure used in this lesson to establish the fact that vertical angles are congruent (deduction: the generalization is made on the basis of a logical argument from facts which have been previously accepted as true).

What type of "proof" do you feel is stronger? Why?

What ideas were assumed to be true in the argument about vertical angles? (Two adjacent angles which form a straight angle are supplementary; two quantities equal to the same quantitity are equal to each other; when equal quantities are subtracted from equal quantities, the differences are equal.)

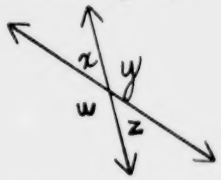
Define Postulate: a statement which is accepted as true, without proof.

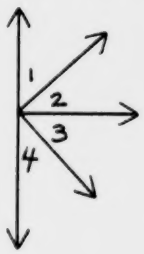
Theorem: a statement which has been proved by deductive or logical reasoning.

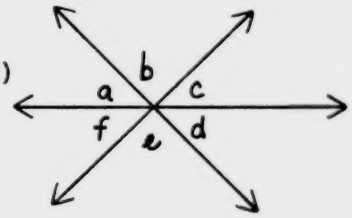
The statement, "Vertical angles are congruent," is a theorem. In a logical system, it is necessary to have some statements which are accepted without proof, just as it is necessary to have some "undefined" terms.

APPLICATIONS:

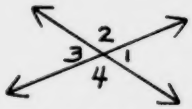
1. Name pairs of vertical angles if any exist (assume lines are straight).

a) 

b) 

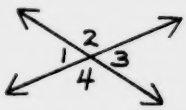
c) 

2. If m∡2 = 125°, find m∡1, m∡3, m∡4.



3. If one angle measures $27\frac{7}{8}°$, what is the measure of its vertical angle?

4. Find the measure of each angle. m∡1 = x + 15, m∡3 = 2x - 10



5. If two vertical angles measure 3y + 20 and 5y, find the measure of each angle.

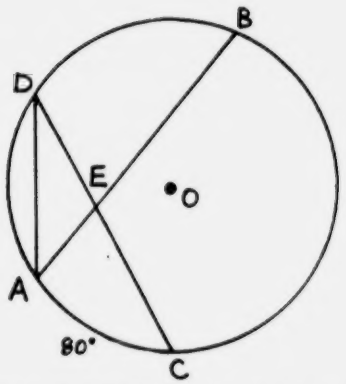
6. Two lines, $\overline{AB}$ and $\overline{CD}$, intersect at E. If m∡AEC = 2x and m∡CEB = 3x + 30, find m∡DEB.

SUMMARY:

1. What are vertical angles?
2. What is true about every pair of vertical angles?
3. If a pair of vertical angles is also complementary, what is the measure of each angle?
4. If a pair of vertical angles is also supplementary, what is the measure of each angle?
5. What differentiates a theorem from a postulate?

HOMEWORK:

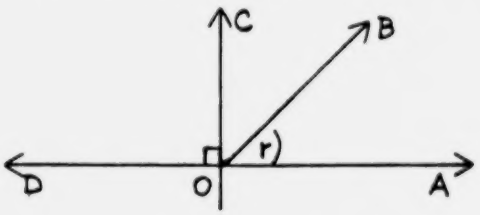
1. In circle O chords $\overline{AB}$ and $\overline{CD}$ intersect E. Chord $\overline{DA}$ is drawn. $m\overset{\frown}{DB} = 70°$. $m\overset{\frown}{AC} = 80°$. Find the number of degrees in



   a) $\measuredangle A$
   b) $\measuredangle D$
   c) $\measuredangle DEB$

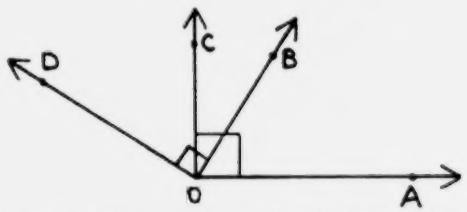
2. If an angle and its vertical angle measure 2x and x + 50 degrees respectively, how large is the angle?

3. Name the angle in the adjacent figure



   a) complementary to $\measuredangle r$
   b) supplementary to $\measuredangle r$

4. In the adjacent figure $\overrightarrow{OA} \perp \overrightarrow{OC}$ and $\overrightarrow{OB} \perp \overrightarrow{OC}$. If $m\measuredangle AOB = 35°$, find



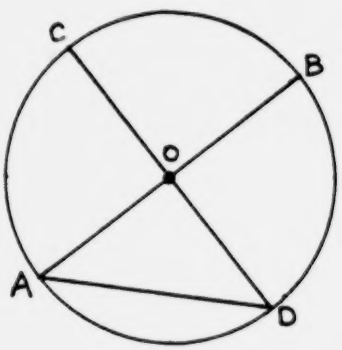
   a) $m\measuredangle BOC$
   b) $m\measuredangle COD$

5. Using the figure for ex. 4, if $m\measuredangle AOB = n°$, find in terms of n
   a) $m\measuredangle BOC$ b) $m\measuredangle COD$

6. In the figure for ex. 4, which two angles have the same complement? Try to formulate a generalization about complements of the same angle.

7. Show that supplements of the same angle must have the same measure.

8. Diameters $\overline{AB}$ and $\overline{CD}$ are drawn in circle O.
   a) Name 2 pairs of equal angles.
   b) Name 2 pairs of equal arcs.



9. In the figure for ex. 8, suppose m∡A = 25°. What is the measure of

   a) $\overset{\frown}{BD}$; d) $\overset{\frown}{AC}$; g) Name 2 diameters.

   b) ∡D; e) ∡AOC; h) Name 4 radii.

   c) ∡AOD; f) ∡AOD. i) Name 1 chord.

10. An angle increased by its complement yields a sum equal to its supplement decreased by 20°. How large is this angle?

11. What are some postulates for:

   a) the operation of addition;

   b) the operation of multiplication;

   c) solving equations;

   d) logic;

   e) geometry.

12. Which statements in mathematics do you find we have proved

   a) in logic?

   b) in arithmetic?

   c) in geometry?

13. Is it possible to prove some postulates? If so, why do we take them to be postulates?

14. Is it possible to prove everything in mathematics? Explain.

Lesson 6 - Geometry

AIM: 1. What are parallel lines?

2. What are the postulates and theorems related to parallel lines?

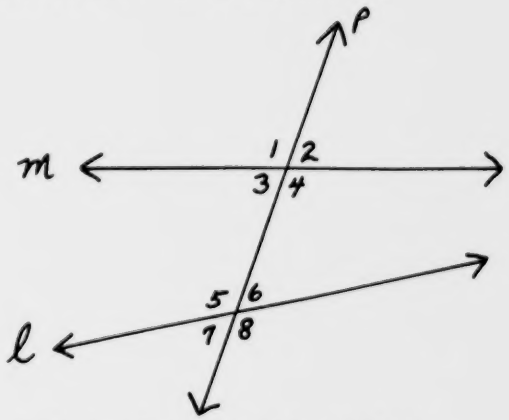
PERFORMANCE OBJECTIVES: The student will be able to...

1. Define transversal.
2. Define alternate interior angles, corresponding angles, interior angles on the same side of the transversal.
3. Define parallel lines.
4. Informally and formally derive the postulates and theorems related to parallel lines.
5. Solve numerical and algebraic problems dealing with parallel lines.

VOCABULARY: transversal, alternate interior, corresponding angles, parallel lines

MOTIVATION:

Name all pairs of angles in this figure which you could prove are congruent on the basis of what we have learned.



DEVELOPMENT:

1. Students should identify four pairs of vertical angles which are congruent. (∡1 and ∡4, ∡2 and ∡3, ∡5 and ∡8, ∡6 and ∡7.)
2. Define: A <u>transversal</u> is a line that intersects two or more lines in distinct points.
   Ask, "Which line in this figure is a transversal?" (line p)
3. Ask: How would you describe the positions of ∡4 and ∡5? (nonadjacent, inside, and on opposite sides of the

transversal) Ask, "What would you call these? (alternate interior angles) Are there any other pairs of alternate interior angles?" (∢3, ∢6)

4. Ask, "How would you describe the positions of ∢1 and ∢5?" (On the same side of the transversal and above the lines ℓ and m (nonadjacent), one is interior and the other is exterior.) These are called corresponding angles. Ask, "Are there other pairs of corresponding angles?" (∢3 and ∢7, ∢2 and ∢6, ∢4 and ∢8)

5. "What would you call ∢3 and ∢5, ∢4 and ∢6?" (Interior angles on the same side of the transversal.)

6. Ask, "Under what circumstances do you think corresponding angles 2 and 6 would be congruent?"
   Students should suggest that this will happen when lines ℓ and m are parallel.
   Elicit the definition: Parallel lines are lines in the same plane that do not intersect. (Notation: $\ell \parallel$ m)
   Ask, "If the lines were parallel, do you believe ∢2 and ∢6 would be the only pair of corresponding angles which are congruent?" (No, if they were congruent, then all pairs of corresponding angles would be congruent.)
   The class should be willing to accept as a postulate: If two parallel lines are cut by a transversal, then the corresponding angles formed are congruent.

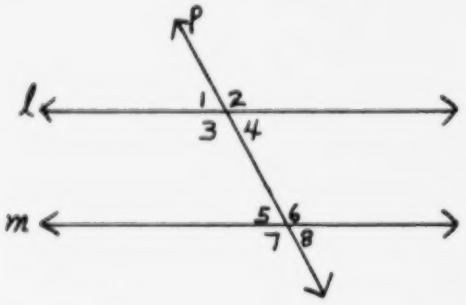
7. Ask, "What conclusion would you draw concerning alternate interior angles when two lines are parallel?" Elicit the statement: When two lines are parallel, the alternate interior angles are congruent (equal in measure).
   Ask, "Can the truth of this statement be shown to follow from the postulate stated earlier?
   How can we demonstrate that ∢3 must be congruent to ∢6?"
   Elicit: By the postulate, ∢6 is congruent to ∢2. ∢2 and ∢3 are congruent because they are vertical angles.
   Therefore, ∢3 must be congruent to ∢6.
   By a similar argument, one might show that ∢4 and ∢5 must be congruent.

"Since the statement about alternate interior angles has been proven by logical reasoning, what may we call it?" (a theorem)

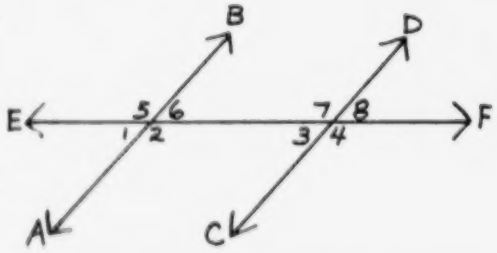
8. Ask: "What is the relationship between m∡4 and ∡3?" (supplementary) "What is the relationship between ∡4 and ∡5?" (Congruent, since they are alternate interior angles.) "What conclusion could be drawn about ∡3 and ∡5?" (They are supplementary.) Elicit the theorem: If two lines are parallel, then the interior angles on the same side of the transversal are supplementary.

APPLICATIONS:

1. $\ell \parallel m$
   If m∡1 = 60°, find the measures of all other angles in the figure.

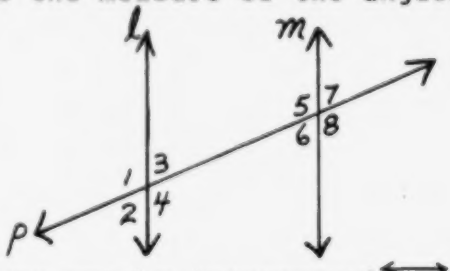


2. $\overleftrightarrow{AB} \parallel \overleftrightarrow{CD}$
   a) If m∡5 = 108°, find the measure of all the other angles.
   b) If m∡4 = 130°, find m∡7. Why?
   c) If m∡1 = 125°, find m∡8. Why?



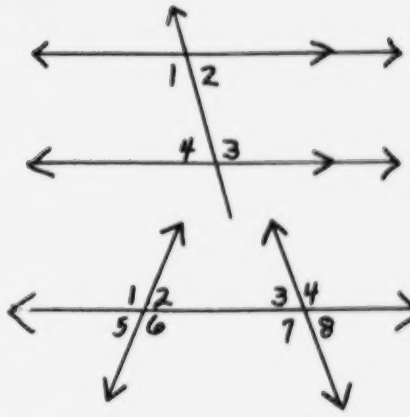
3. When two lines are parallel, one alternate interior angle measures $(x + 30)°$ and the other measures $(3x - 10)°$. Find the value of x. What is the measure of the angles?

4. In this figure $\ell \parallel m$. If the measure of ∡2 = 2x + 8 and the m∡6 = 5x - 13, find the measure of all eight angles.



5. Parallel lines $\overleftrightarrow{AB}$ and $\overleftrightarrow{CD}$ are cut by transversal $\overleftrightarrow{EF}$, such that $\overleftrightarrow{AB}$ intersects $\overleftrightarrow{EF}$ at G, and $\overleftrightarrow{CD}$ intersects $\overleftrightarrow{EF}$ at H. If m∡AGE = 3y + 11 and the m∡DHF = 6y - 9, find y.

6. If the m∡1 = 6x - 10 and m∡4 = 4x, find the measure of the four angles.



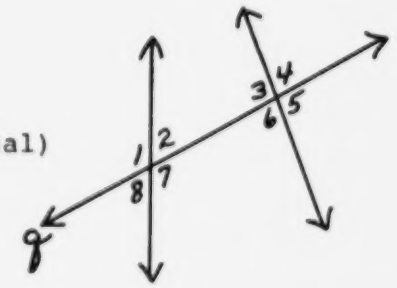
7. a) Name pairs of alternate interior angles.
   b) Name pairs of corresponding angles.
   c) Name pairs of alternate exterior angles.
   d) Name pairs of interior angles on the same side of the transversal.

SUMMARY:

1. What are parallel lines?
2. What is a transversal?
3. What are corresponding angles?
4. When two lines are cut by a transversal, how many pairs of alternate interior angles are formed?
5. Which postulate and theorems of parallel lines can we now state?

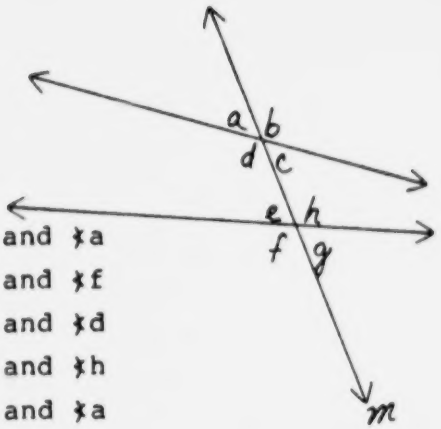
HOMEWORK:

1. For the figure at the right, name 2 pairs of angles that are: (q is the transversal)
   a) corresponding.
   b) alternate interior.
   c) alternate exterior.
   d) interior angles on the same side of the transversal.
   e) exterior angles on the same side of the transversal.
   f) equal in measure. Can you name two more pairs?
   g) supplementary.



2. Describe the following pairs of angles:
(The transversal is m.)

a) ∡a and ∡e
b) ∡a and ∡g
c) ∡c and ∡e
d) ∡c and ∡g
e) ∡c and ∡h
f) ∡c and ∡b
g) ∡c and ∡a
h) ∡b and ∡f
i) ∡b and ∡d
j) ∡b and ∡h
k ∡b and ∡a

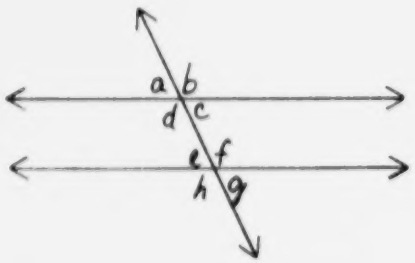


3. In the figure for ex. 2, suppose m∡a = 70° and m∡e = 80°. Find the measures of:

a) ∡g
b) ∡b
c) ∡c
d) ∡d
e) ∡h
f) ∡f

4. If ℓ // m, what reason would you give for saying:

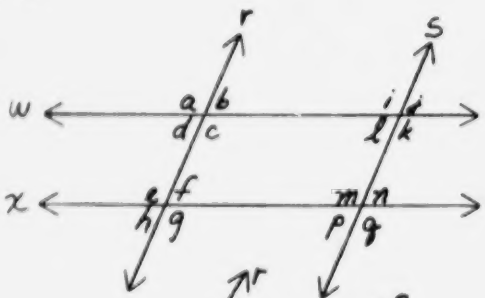
a) m∡a = m∡e
b) m∡c = m∡e
c) m∡c = m∡g
d) m∡b = m∡f
e) m∡d = m∡f



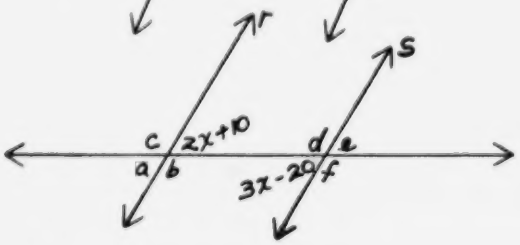
5. Using the information in ex. 4 and m∡a = 50°, find the measures of: b, c, d, e, f, g.

6. We are given that w // x and r // s. Find all of the angles congruent to

a) a.
b) b.



7. In the figure at the right with r // s, find the degree measure of: a, b, c, d, e, f.



8. In the figure at the right diameter $\overline{AOB}$ is in a line parallel to $\overline{DC}$, and $m\widehat{AD} = 80°$.

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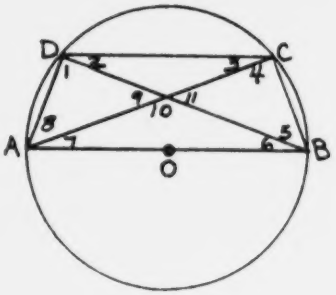
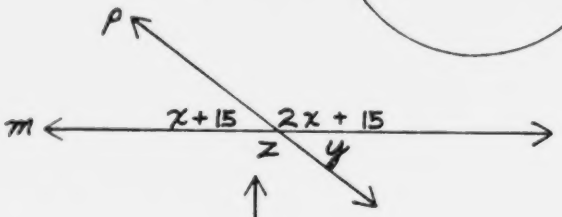
Find the measure of:

a) ∡b
b) ∡2
c) $\overset{\frown}{CB}$
d) $\overset{\frown}{DC}$
e) ∡5
f) ∡8
g) ∡1
h) ∡4



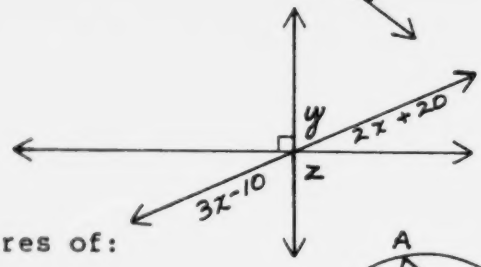
9. Solve for

a) x
b) y
c) z



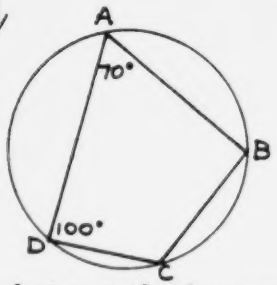
10. Solve for:

a) x
b) y
c) z



11. Find the measures of:

a) ∡C
b) ∡B
c) $\overset{\frown}{ABC}$
d) $\overset{\frown}{BCD}$
e) $\overset{\frown}{AB}$ if $\overset{\frown}{BC}$ = 80°.



12. State the converses of the postulate and theorems discussed today. Do you believe the converses are true?

Lesson 7 - Geometry

AIM: What theorem can be stated regarding the measures of the angles of a triangle?

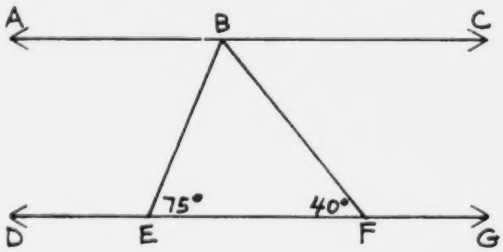
PERFORMANCE OBJECTIVES: The student will be able to...

1. State the sum of the angles theorem.
2. Informally and formally prove the theorem.
3. Solve numerical and algebraic problems involving the theorem.

VOCABULARY: auxiliary line

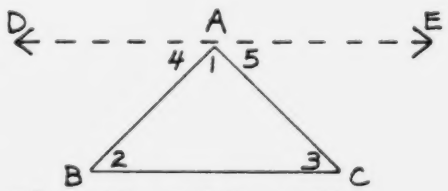
MOTIVATION:

1. If $\overleftrightarrow{ABC} // \overleftrightarrow{DEFG}$, how many degrees are there in ∡EBF?



DEVELOPMENT:

1. Students may want to use the alternate interior angles theorem to find the measures of ∡ABE and ∡CBF. Since the three angles with common vertex B must add up to 180°, the measure of ∡EBF may be found by subtraction (65°).
2. Ask: "What is the sum of the measures of the angles in triangle BEF?" (180°)
3. Give different values for the measures of ∡E and ∡F and repeat as above. Observe that the sum is again 180°. Ask: "Will the sum of the angles always be 180°?" Elicit the statement: The sum of the measures of the angles of a triangle equals 180°.
4. Ask: "How can we prove that this is true in the general case?" Elicit: Draw a triangle on the chalkboard and number the angles.



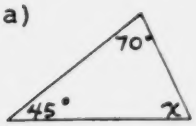
Ask: "What are we trying to prove?" (m∡1 + m∡2 + m∡3 = 180°).

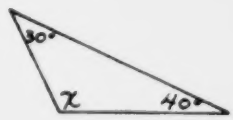
Ask: "What else is needed in the figure to enable us to imitate the reasoning used earlier?"
Draw a line DE through A parallel to line BC. Number the two new angles formed as 4 and 5. "What is true about m∢4 + m∢1 + m∢5?" (They form a straight angle, therefore they add up to 180°. m∢4 + m∢1 + m∢5 = 180°.) Elicit that ∢4 and ∢2 are alternate interior angles as are ∢3 and ∢5. "What is true about their measures?" (They are equal.) "Therefore if we replace m∢4 with m∢2 and m∢5 with m∢3, what statement do we have?" (m∢1 + m∢2 + m∢3 = 180°)

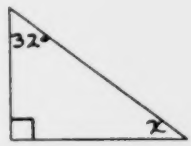
5. Ask: Now that we have proved the statement about the sum of the measures of the angles in a triangle, what are we entitled to call this statement? (a Theorem)

APPLICATIONS:

1. Find the third angle in each triangle.

a) 

b) 

c) 

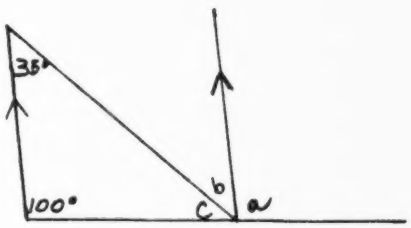
2. Find the measure of the third angle in a triangle if two angles measure:
   a) 110°, $25\frac{1}{2}$° b) 38.5°, 59.4° c) 95°, 100° (no triangle)
3. In a right triangle one of the angles measures 25°. What are the measures of the other two angles?
4. If the three angles of a triangle measure 2x, 2x and 5x, what are their numerical measures?
5. In a right triangle, one acute angle measures 2x - 20 and another acute angle measures x + 10. Find the measures of the three angles of the triangle.
6. If the first angle of a triangle is twice as large as the second angle and the third angle is 20 more than the first, find the measure of each of the three angles.

SUMMARY:

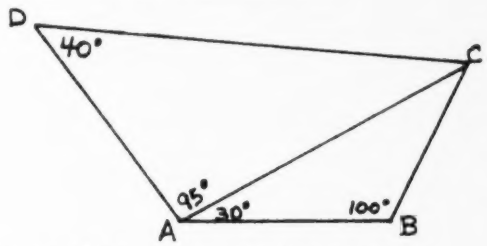
1. What is the theorem relating the measures of the angles of a triangle?
2. What earlier statements were needed to prove this theorem?

HOMEWORK:

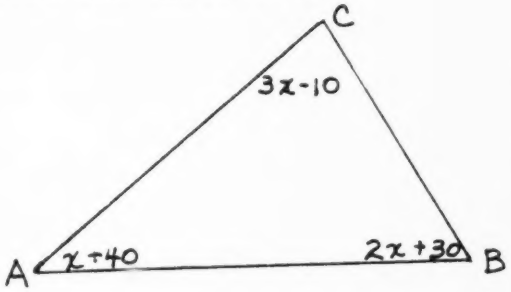
1. Using the figure at the right, find the measure of:
   a) ∡a
   b) ∡b
   c) ∡c



2. Using the figure at the right, find the measure of all 4 angles of the quadrilateral ABCD. What is the sum for all 4 angles?



3. The ratio of the measures of the angles of a triangle is 2:3:5. What are their measures?
4. If the triangle is ex. 3 is inscribed in a circle, what is the measure of each of the three arcs?
5. For the triangle at the right find the measures of all 3 angles.



Lesson 8 - Geometry

AIM: What is a quadrilateral?

PERFORMANCE OBJECTIVES: The student will be able to...

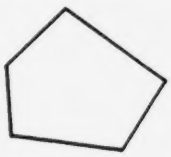
1. Define quadrilateral.
2. Classify quadrilaterals as a parallelograms, rhombi (or rhombuses), rectangles, squares, trapezoids.
3. Define polygon.

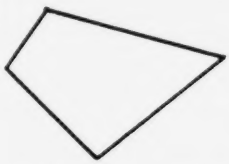
VOCABULARY: polygon, quadrilateral, parallelogram, rhombus, rectangles, squares, trapezoid

MOTIVATION:

What do these figures have in common? How do they differ?

a)

b) 

c) 

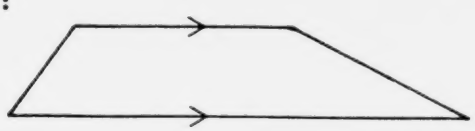
DEVELOPMENT:

1. What do all three figures have in common? Elicit: They are all closed figures made up of line segments. State: They are all polygons. Define: A polygon is a simple closed figure made up of line segments.
2. Elicit that polygons may be classified according to number of sides. Elicit that c) has four sides. Ask, "What is a prefix that means four?" (quad, as in quadrophonic sound) State that figure c) is a quadrilateral. Ask, "What is a quadrilateral?" Definition: A quadrilateral is a polygon with four sides.
3. Students will have had some experience with various special quadrilaterals.
   Ask, "Can you name any quadrilaterals which are special?"
   Ask students to draw the figures they name and to find examples in the room if possible.
   Elicit the definitions of parallelogram, rectangle and rhombus.

A <u>parallelogram</u> is a quadrilateral having parallel opposite sides. Its symbol is $\square$, as in $\square$ABCD.
A <u>rectangle</u> is a parallelogram with a right angle. (Note: <u>One</u> right angle is sufficient; the parallel lines force the other angles to be right angles also.
A <u>rhombus</u> is a parallelogram with congruent adjacent sides.

Ask: What other quadrilateral has congruent adjacent sides? (a square) What differentiates a square from a rhombus? Elicit the definition: A <u>square</u> is a rhombus with a right angle. Could there be another definition? A <u>square</u> is a rectangle with congruent adjacent sides.

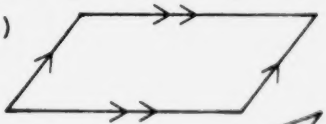
4. Consider this figure:



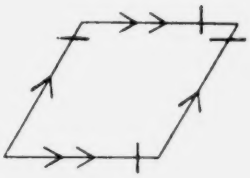
How would you describe this figure? Elicit that it has only one pair of parallel sides. State that this is a trapezoid. Elicit the definition: A <u>trapezoid</u> is a quadrilateral with only one pair of parallel sides.

APPLICATIONS:

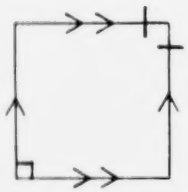
1. Identify each figure.

a) 

b)

c) 

d)

e) 

2. True or false?
   a) All trapezoids are quadrilaterals.
   b) All squares are rectangles.
   c) All rectangles are squares.
   d) All squares are rhombuses.
   e) All rhombuses are squares.
   f) All trapezoids are parallelograms.

3. Which of the following have <u>congruent</u> diagonals?

   a) parallelogram
   b) rectangle
   c) rhombus
   d) square
   e) trapezoid

4. In which (if any) of the special quadrilaterals do you think that:

   a) the diagonals will be perpendicular?
   b) the pairs of opposite sides will be congruent?

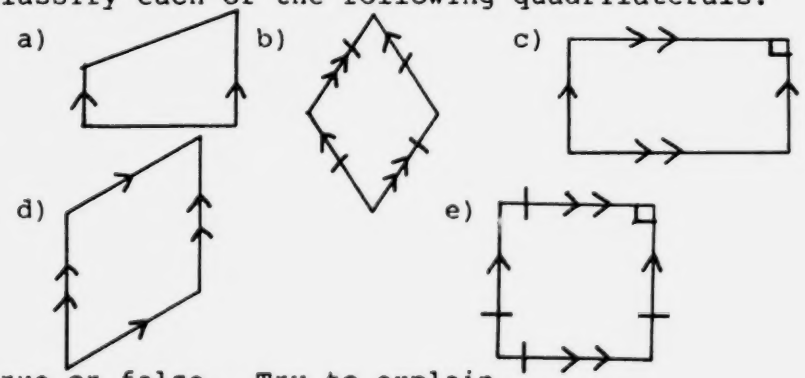
SUMMARY:

Consider the four sets: {Parallelograms}, {Rectangles}, {Rhombuses}, {Squares}.

1. Which set contains each of the other three?
2. Which set is contained in each of the other three?

HOMEWORK:

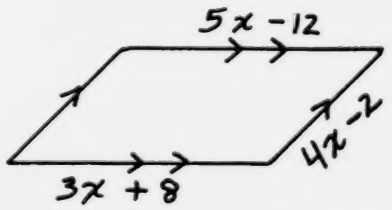
1. Classify each of the following quadrilaterals.



2. True or false. Try to explain.

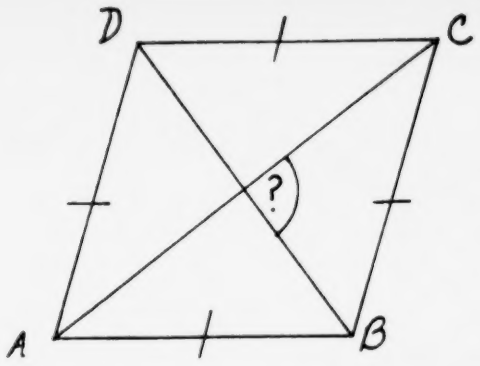
   a) A trapezoid must have at least one acute and one obtuse angle.
   b) A parallelogram must have at least one acute and one obtuse angle.

3. Three sides of a parallelogram have lengths as shown. Find the lengths of all 3 sides and decide whether it is a rhombus.



(Note: Opposite sides in a parallelogram are congruent.)

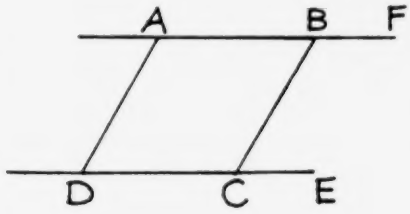
4. Draw carefully a rhombus ABCD and diagonals $\overline{AC}$ and $\overline{BD}$. Using your protractor, measure the angle made by the two diagonals. Conjecture a generalization.



5. Draw two line segments, $\overline{AB}$ and $\overline{CD}$, that bisect each other. Connect their endpoints to form a quadrilateral. Conjecture the type of quadrilateral we have.

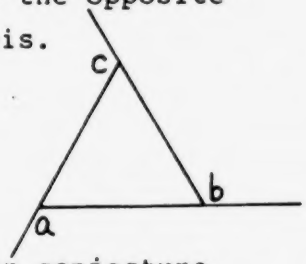
6. Draw two line segments, $\overline{AB}$ and $\overline{CD}$, that bisect each other (cut each other in half) and are perpendicular. Connect their endpoints. Conjecture the kind of quadrilateral we have.

7. $\square$ABCD has m∡BAD = 130°.
Find the degree measure of:
   a) ∡CBF
   b) ∡CBA
   c) ∡BCE
   d) ∡BCD
   e) ∡ADC



8. What seems to be true about the measures of the opposite angles of a parallelogram? Try to prove this.

9. Draw any triangle and extend the sides as shown. Find m∡a + m∡b + m∡c. What conjecture might you make about all such sums? Try to prove your conjecture.



10. Draw any quadrilateral. Join the midpoints of consecutive sides. What type of quadrilateral is formed?

11. Draw a rectangle. Join the midpoints of consecutive sides. What type of quadrilateral is formed?

12. Draw a rhombus. Join the midpoints of consecutive sides. What type of quadrilateral is formed?

Lesson 9 - Geometry

AIM: What is the sum of the measures of the angles of a quadrilateral?

PERFORMANCE OBJECTIVES: The student will be able to...

1. Recall that the sum of the angles of a triangle is 180°.
2. State that the sum of the measures of the angles of a quadrilateral is 360°.
3. Solve numerical and algebraic problems involving the measures of the angles of a quadrilateral.

VOCABULARY: quadrilateral

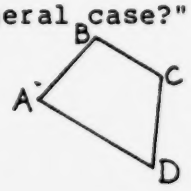
MOTIVATION:

For each of the following, draw the figure and tell what the sum of its angles would be.

1. rectangle 2. parallelogram 3. trapezoid

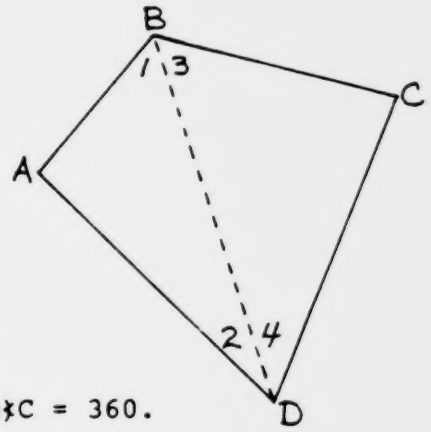
DEVELOPMENT:

1. Discuss motivation. Students should observe that the sum in a rectangle is 4(90°) or 360°.
   The following argument applies for a parallelogram or a trapezoid: since same-side interior angles are supplementary, the sum is 2(180°) or 360°.
2. Ask: "Will the result be the same for <u>every</u> quadrilateral?"
   If the students suggest it will not hold in the general case, challenge them to draw a quadrilateral which has a different sum on the chalkboard; measure the angles with a protractor and verify that the sum is 360°.
3. It seems that: The sum of the measures of the angles in any quadrilateral is 360°.
   Ask: "How can we prove this result in the general case?"
   Draw a general quadrilateral ABCD and state the theorem. ($m\angle A + m\angle B + m\angle C + m\angle D = 360$)



If necessary, ask: How can we relate this problem to the problem of the triangle which we have already solved? (Elicit: by adding an auxillary line (a diagonal) we may divide the quadrilateral into two triangles.)

Elicit that the sum of the four angles in quadrilateral ABCD will equal the sum of the six angles in △ BAD and △BCD, since ∡B = ∡1 + ∡3 and ∡D = ∡2 + ∡4.



In △ BAD, m∡1 + m∡2 + m∡A = 180.

In △BCD, m∡3 + m∡4 + m∡C = 180.

Adding equals to equals, we find

m∡1 + m∡2 + m∡3 + m∡4 + m∡A + m∡C = 360.

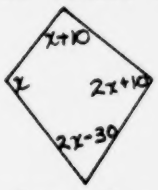
So (making replacements as explained above),

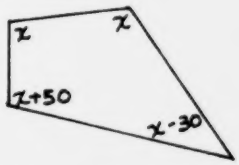
m∡A + m∡B + m∡C + m∡D = 360.

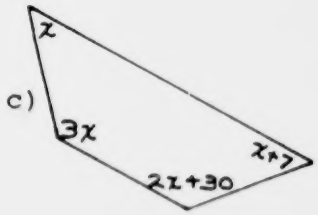
The statement has been proved, and may be regarded as a <u>theorem</u>.

APPLICATIONS:

1. Find the missing angle given three angles of a quadrilateral as follows.
   a) 110°,70°,60° b) 58°,77°,29°
   c) 70.5°,83.2°,16.3°

2. Find the value of x in each figure.
   a) 

   b) 

   c) 

3. Two angles of a quadrilateral are equal in measure. A third angle measures 50° more than these. The fourth angle measures 30° more than the equal angles. Find the measure of each angle.

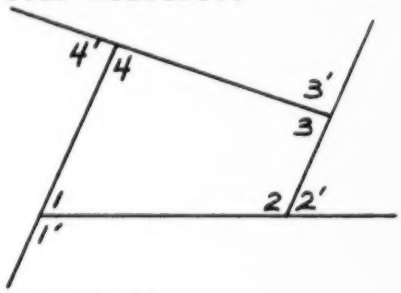
4. In the quadrilateral ABCD the measure of ∡A is twice the measure of ∡B, the measure of ∡C is 10° less than twice the measure of ∡A and the measure of ∡D is 30° more than the measure of ∡A. Find all the angles.

SUMMARY:

1. How many straight angles would be needed to give the same sum as the measures in the four angles of a quadrilateral?
2. What is the sum of the measures of the angles of a pentagon?

HOMEWORK:

1. The ratio of the measures of the angles of a quadrilateral is 3:4:5:6. Find all four measures.
2. Suppose:
   $m\angle 1 = m\angle 2 - 10$
   $m\angle 3 = m\angle 2 + 10$
   $m\angle 4 = 2m\angle 1$
   Find the measures of 1,2,3,4.



   Find the sum of the measures of 1'+2'+3'+4'.
   Conjecture a generalization about such sums for quadrilaterals. Try to prove your conjecture.
3. Show that if $\square$ABCD is inscribed in a circle, it must be a rectangle.
4. Draw a quadrilateral having
   a) 4 right angles
   b) 3 acute angles
   c) 3 obtuse angles
   d) 2 right angles and an obtuse angle.
   e) Can a quadrilateral have 4 acute angles? Explain.
   f) Can a quadrilateral have 4 obtuse angles? Explain.

Lesson 10 - Geometry

AIM: What are the properties of a parallelogram?

PERFORMANCE OBJECTIVES: The student will be able to...

1. State the properties of a parallelogram.
2. Solve numerical and algebraic problems involving the properties of a parallelogram.

VOCABULARY: opposite sides, opposite angles, consecutive angles diagonal, bisect

MOTIVATION:

Using a protractor and a ruler, draw a parallelogram.

DEVELOPMENT:

1. Ask, "What is a parallelogram?" (quadrilateral with both pairs of opposite sides parallel) Discuss the motivation problem. It is easy to draw the first two sides (two intersecting lines). We now have three vertices of the desired parallelogram. How can we place the next line so that it is parallel to one of these sides?
   Elicit that special relationships exist between the angles formed when parallel lines are cut by a transversal. Students should suggest using the protractor to measure the angle formed between the first two sides and then to draw a congruent angle in a "corresponding" or "alternate interior" position.
   Expecting this strategy to produce a pair of lines which are parallel is relying on the truth of the converse of a statement about parallel lines established in a previous lesson. Although some true statements have false converses, each of the true statements about angles formed when parallel lines are cut by a transversal has a converse which is true (if one of them is postulated as true, it can be used to prove the others). The protractor may be used again to place the fourth side of the parallelogram. Label the vertices of $\square$ABCD.

2. Ask, "Since in a parallelogram opposite sides are parallel, what is true about ∡A and ∡D?" Note: They are interior angles on the same side of transversal $\overline{AD}$, therefore, they are supplementary.
   Using similar reasoning, show all pairs of consecutive angles are supplementary and elicit Property 1: Consecutive angles of a parallelogram are supplementary.

3. Ask, "What is the relationship between opposite angles in a parallelogram?" Students should observe that they are congruent (measure them). "Will this always be true?" Elicit: Since ∡A is supplementary to ∡B and ∡C is supplementary to ∡B, ∡A and ∡C must be congruent. Record Property 2: The opposite angles of a parallelogram are congruent (equal in measure).

4. Ask, "Do you observe any other properties which every parallelogram must have?" Students should observe that opposite sides have equal lengths (measure). Record Property 3: The opposite sides of a parallelogram are congruent (equal in length). (This may be proven later in the course, after other ideas have been established. Meanwhile, the class should accept its truth on the teacher's authority or as a postulate.)
   Are the diagonals congruent? (Clarify the definition if necessary: line segment joining nonconsecutive vertices of a polygon.) Draw both diagonals and measure their lengths. Ask if any student found the diagonals of his parallelogram to be congruent. "Is it possible for the diagonals of a parallelogram to be congruent?" (Yes, in a rectangle or a square.)
   Suggest that the students measure the lengths of the segments of the diagonals.
   Elicit Property 4: The diagonals of a parallelogram bisect each other. (Students do not have sufficient information to prove this now, but it is true.)

APPLICATIONS:

The properties of a parallelogram may be used to find the measures of unknown parts.

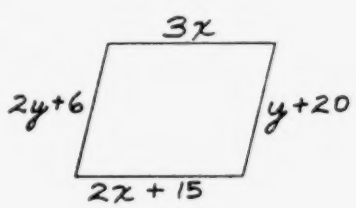
1. Find the measure of the angles of a parallelogram if one angle measures
   a) 100° b) 75° c) 93.5°
2. In parallelogram ABCD, the measure of ∡A equals 3x + 10 and the measure of ∡C equals 2x + 40. Find the measure of ∡A.
3. The diagonals of parallelogram NORA intersect at point H. If HO = 5x + 2 and HA = 3x + 10, find the length of $\overline{OA}$.
4. In parallelogram WXYZ, the measure of ∡W equals 3x - 8 and the measure of ∡X equals 4x + 15. Find the measure of all the angles of the parallelogram.
5. In parallelogram FAST, FA equals 3x - 9 and ST equals 2x + 17. Find the value of x.
6. In parallelogram FAST, the measure of ∡F is 8 more than 3 times the measure of ∡A. Find the measure of all the angles of this parallelogram.

SUMMARY:

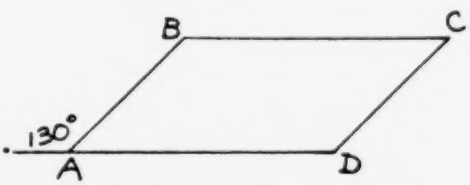
1. What is true about the angles of a parallelogram?
2. What is true about the sides of a parallelogram?
3. What is true about the diagonals of a parallelogram?
4. Why is it useful to be aware of these properties?

HOMEWORK:

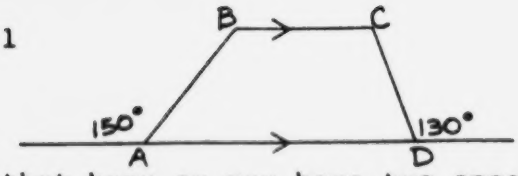
1. Find the perimeter of the parallelogram at the right. A numerical answer is sought.



2. Find the measures of all the angles of the parallelogram at the right.



3. Find the measure of all the angles of the trapezoid.



4. Name 4 quadrilaterals that have or may have two opposite sides that are congruent.

5. What special quadrilateral may have no two sides congruent?

6. If one angle of a parallelogram is 4 times some other angle, what are their measures?

Lesson 11 - Geometry

AIM: When is a quadrilateral a parallelogram?

PERFORMANCE OBJECTIVES: The student will be able to...

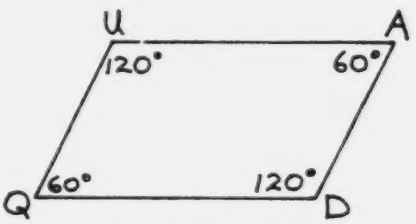
1. State conditions sufficient that a quadrilateral be a parallelogram.
2. Recognize a parallelogram from a marked diagram.

VOCABULARY: no new terms

MOTIVATION:

The measure of the angles of a quadrilateral, in order, are $2x$, $4x$, $x + 30$ and $5x - 30$. What kind of quarilateral is it?

DEVELOPMENT:



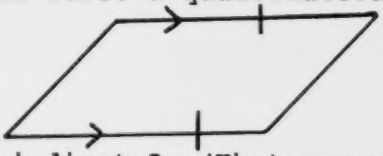
1. Discuss motivation. Students should recall that the sum of the measures of the angles in any quadrilateral is 360°. This leads to the equation $12x = 360$. Therefore $x = 30°$ and the angles, in order, are 60°, 120°, 60° and 120°. Students should observe that the opposite angles are congruent.
   "What kind of quadrilateral is it?" Elicit that we have previously established that, in a parallelogram, the opposite angles are congruent. (If a quadrilateral is a parallelogram, then the opposite angles in the quadrilateral are congruent.) Is the converse of this statement true?
   Elicit that, by the definition, to demonstrate that a quadrilateral is a parallelogram, we must establish that <u>both</u> pairs of opposite sides are parallel. (List as <u>Method 1</u>: If both pairs of opposite sides are congruent, then the quadrilateral is a parallelogram.)

In this case, we know that both pairs of opposite angles are congruent. Is this sufficient to guarantee that both pairs of opposite sides will be congruent? Students should agree that it is sufficient, since the class has previously accepted as true the idea that two supplementary same-side interior angles will guarantee parallel lines.

(List as Method 2: If both pairs of opposite angles are congruent, then the quadrilateral is a parallelogram.)

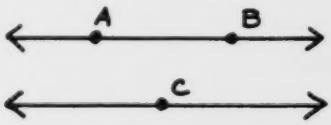
2. Ask, "Do you think that there are any other sets of conditions which would force a quadrilateral to be a parallelogram?"

Consider the figure:



What do the markings indicate? (That one pair of opposite sides is both parallel and congruent.) Ask, "Do you think that this quadrilateral must be a parallelogram?"

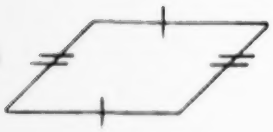
State: Let's try to draw a figure that fits these conditions. Using your ruler and protractor, draw two parallel lines. Pick two points on one of the lines (A,B). Pick a point C on the second line and measure the distance between points A and B. Find point D to the right of C on the second line such that $\overline{AB}$ is congruent with $\overline{CD}$. Now connect $\overline{AC}$ and $\overline{BD}$. What kind of figure do we have? Students should use rulers and protractors to confirm that the figures they draw have all the properties of a parallelogram.

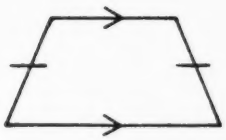


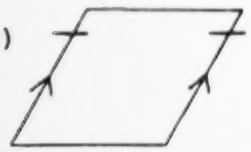
Elicit Method 3: If one pair of opposite sides of a quadrilateral is both parallel and congruent, the quadrilateral is a parallelogram.

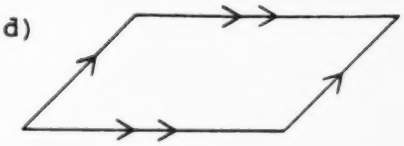
APPLICATIONS:

1. Which of the following are parallelograms? (If à parallelogram, state the reason.)

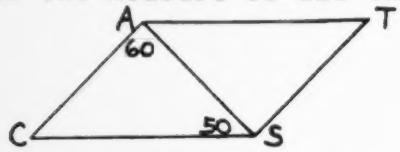
a) 

b) 

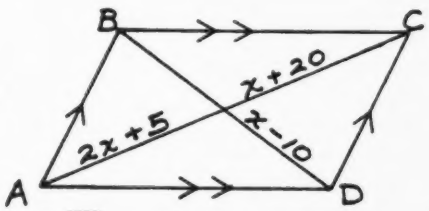
c) 

d) 

2. Find the measure of all the angles of the parallelogram.



3. a) Find the value of x.
   b) Find the lengths of $\overline{BD}$ and $\overline{AC}$.



4. In parallelogram WXYZ, $\overline{WY}$ and $\overline{XZ}$ meet in point O. If WO = 3y - 10 and OY = 2x + 10, find the length of $\overline{WY}$.

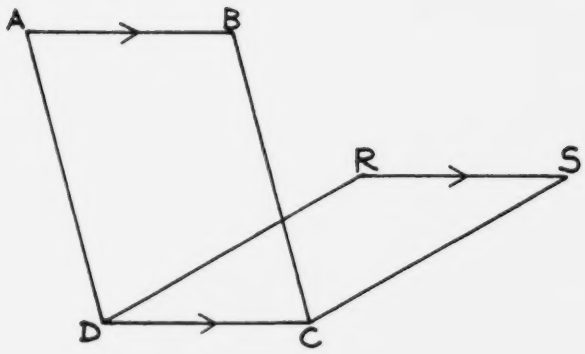
SUMMARY:

1. When is a quadrilateral a parallelogram? (State three sets of conditions, each of which being met would guarantee that the quadrilateral is a parallelogram.)
2. Why is it useful to know this? (If you recognize that a figure is a parallelogram, then you know that it has special properties. This information can be useful!)

HOMEWORK:

1. In quadrilateral ABCD, $\overleftrightarrow{AB} \parallel \overleftrightarrow{DC}$, AB = DC. Regarding this figure, state:
   a) The kind of quadrilateral ABCD is.
   b) A relation between consecutive angles.
   c) Three equalities regarding its sides and angles.
   d) A relation between $\overline{AD}$ and $\overline{BC}$ not previously mentioned.
2. In the figure described above, if AD = 7", what must the length of $\overline{BC}$ be? Explain.

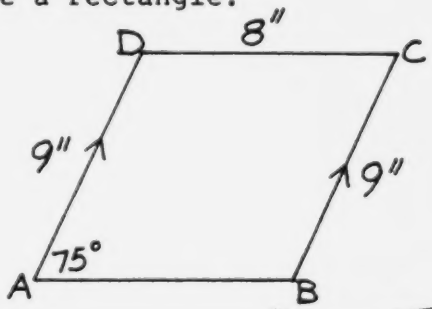
3. In the figure at the right $\overline{AB} \parallel \overline{DC}$, $\overline{RS} \parallel \overline{DC}$, AB = DC, RS = DC.
What relation holds between:



a) $\overline{AB}$ and $\overline{RS}$. Explain.
b) AB and RS. Explain.
c) $\overline{AD}$ and $\overline{BC}$. Explain.
d) AD and BC. Explain.
e) $\overline{RD}$ and $\overline{SC}$. Explain.
f) RD and SC. Explain.

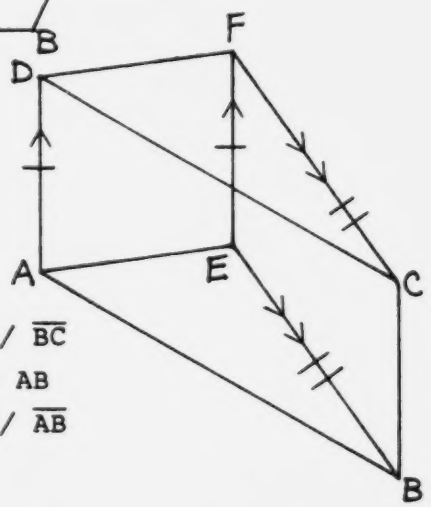
4. Answer TRUE or FALSE and explain:
a) If $\overleftrightarrow{AB} \parallel \overleftrightarrow{CD}$, $\overline{AB}$ and $\overline{CD}$ are in the same plane.
b) If $\overleftrightarrow{AB}$ and $\overleftrightarrow{CD}$ do not intersect, they must be parallel.
c) If a quadrilateral has 2 parallel sides and 2 congruent sides, then it must be a $\boxed{P}$.
d) If all 3 angles of a quadrilateral are right angles, the fourth angle must also be a right angle.
e) If all 4 angles of a quadrilateral are congruent, then it must be a rectangle.

5. Find the measure of the following:



a) ∡B
b) ∡D
c) ∡C
d) BC

6. In the figure shown at the right, explain why



a) AEFD is a $\boxed{P}$
b) EFCB is a $\boxed{P}$
c) $\overline{EF} \parallel \overline{BC}$
d) EF = BC
e) $\overline{EF} \parallel \overline{AD}$
f) EF = AD
g) AD = BC
h) $\overline{AD} \parallel \overline{BC}$
i) DC = AB
j) $\overline{DC} \parallel \overline{AB}$

Lesson 12 - Geometry

AIM: What are the properties of a rectangle, square and rhombus?

PERFORMANCE OBJECTIVES: The student will be able to...

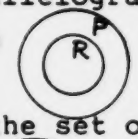
1. List the properties of a rectangle, square and rhombus.
2. Solve numerical and algebraic problems involving the properties of a these quadrilaterals.

VOCABULARY: rectangle, square, rhombus

MOTIVATION:

What is the relationship between parallelograms and rectangles?

DEVELOPMENT:

1. Discuss the motivation problem.
   Elicit the definition of a rectangle: a parallelogram with one right angle. Every rectangle is a parallelogram and enjoys all the properties of a parallelogram. The set of all rectangles is contained with the set of all parallelograms.
   

   There is a larger set, which contains the set of all parallelograms. Students should realize that the larger set is {all quadrilaterals}.

   Ask: "If a rectangle has one right angle, what will be true of the other angles?" Elicit: Since it is a parallelogram, opposite angles are congruent and consecutive angles are supplementary. For rectangles, we have Property 1: In a rectangle all angles are right angles.
2. Draw a rectangle with two diagonals. Ask: "What do we know about these diagonals?" (They bisect each other, because a rectangle is a parallelogram.) Have the students measure the diagonals of the rectangles. Ask: What is true about these diagonals that wasn't true about

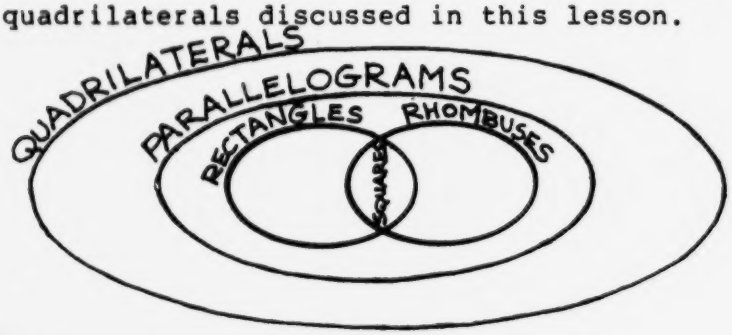
the diagonals of other parallelograms?" Elicit Property 2 of rectangles: The diagonals of a rectangle are congruent. Use a piece of string to illustrate that this property is true of various rectangular objects in the classroom.

3. Review: "What is a rhombus?" (a parallelogram having adjacent sides that are congruent) Observe that the set of rhombuses is contained within the set of all parallelograms. Ask: "What must be true about its opposite sides?" (They must be congruent, since a rhombus is a parallelogram.) Ask: "What special statements can be made about the sides of a rhombus?" Elicit Property 1 of a rhombus: In a rhombus all four sides are congruent. Students should now draw a rhombus, using ruler and protractor. Add the diagonals to the figure. Ask: "Do the diagonals of a rhombus have any special properties?" Elicit: They are (usually) not congruent.
   Elicit Property 2 of a rhombus: The diagonals of a rhombus are perpendicular to each other.
   Elicit Property 3 of a rhombus: The diagonals of a rhombus bisect the angles of a rhombus.
   These properties of a rhombus may be dynamically illustrated by cutting out the rhombus and folding it along the diagonals: fold once on a diagonal to illustrate the same property for the second pair of opposite angles; to illustrate perpendicularity, fold on one diagonal and then (without opening) fold on the second diagonal.

4. Ask: "What is the relationship of a square to each of the above?" Elicit that a square is all of them; it is a parallelogram, a rectangle and a rhombus. Therefore, a square has all the properties previously discussed. Elicit and list all the properties of a square.
   1) The sum of the measures of its angles is 360°.
   2) Opposite sides are parallel.
   3) Opposite sides are congruent.
   4) Opposite angles are congruent.

5) Diagonals bisect each other.
6) All angles are right angles.
7) Diagonals are congruent.
8) All sides are congruent.
9) Diagonals are perpendicular.
10) Diagonals bisect the angles of the quadrilateral.

5. Complete the diagram begun earlier, showing all special quadrilaterals discussed in this lesson.



APPLICATIONS:

1. In rectangle ABCD diagonals $\overline{AC}$ and $\overline{BD}$ are drawn and meet at point E. If the measure of $\overline{AE}$ is 8, find the measure of $\overline{BD}$.

2. Parallelogram ABCD is a rhombus. If AB = 3x + 10 and BC = 5x - 20, find CD.

3. WXYZ is a square. If WY = 3a - 9 and XZ = a + 12, find XZ.

4. LMNO is a rhombus. If the measure of $\measuredangle L = 2x - 9$ and the measure of $\measuredangle M = 3x + 14$, find the measure of all angles of the rhombus.

5. For each of the following answer true or false.
   a) All squares are rectangles.
   b) All squares are rhombuses.
   c) All rhombuses are rectangles.
   d) All rhombuses are squares.
   e) All quadrilaterals are parallelograms.

SUMMARY:

1. What is the the most special quadrilateral? (a square)

2. Examine the list of properties of a square. Which of its properties are due to the fact that it is a quadrilateral? (#1) Which if its properties are due to the fact that it is a parallelogram? (#2,3,4,5) ...a rectangle? (#6,7) ...a rhombus? (#8,9,10)

HOMEWORK:

1. If $m\angle A = 113°$ in $\boxed{P}$ ABCD, find the measures of the other three angles.
2. In rhombus ABCD, side $\overline{AB}$ = diagonal $\overline{AC}$ = 9". Find the measures of:
   a) The other sides of the rhombus.
   b) The 4 angles of the rhombus.
3. In rectangle ABCD, AB = 3x + 2, BC = 2y + 7, CD = 6x - 10, DA = 4y. What is the measure of each side? What else can be said about this figure?
4. Every rectangle is a parallelogram. Is this statement true? State the converse of this statement. Is the converse true? (If yes, explain why. If not, give an example which proves the statement is false.)
5. If a quadrilateral is not in a plane, we say that it is a skew quadrilateral. Admitting skew quadrilaterals, is it possible for such a quadrilateral to have
   a) each of its four angles 30°? Using toothpicks, show such a quadrilateral if possible.
   b) only three of its four angles 90° and the fourth an acute angle? Show such a quadrilateral with toothpicks if possible.
   c) same as b but the fourth angle obtuse?

Lesson 13 - Geometry

AIM: What are the properties of a trapezoid?
What is an isosceles trapezoid?

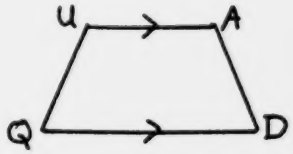
PERFORMANCE OBJECTIVES: The student will be able to...

1. List the properties of a trapezoid.
2. Define an isosceles trapezoid.
3. List the properties of an isosceles trapezoid.
4. Solve numerical and algebraic problems involving the properties of trapezoids.

VOCABULARY: trapezoid, bases, legs, isosceles trapezoid

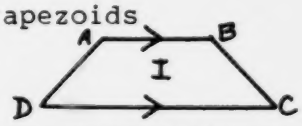
MOTIVATION:

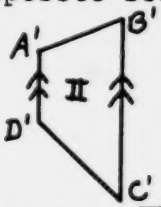
Is QUAD a trapezoid?



DEVELOPMENT:

1. Discuss motivation. QUAD may be a parallelogram or it may not be a parallelogram. If $\overline{QU} \parallel \overline{AD}$, then QUAD is a parallelogram. If $\overline{QU}$ is not parallel to $\overline{AD}$, then QUAD is not a parallelogram. In this case, the figure is called a trapezoid.
   Elicit the definition of a <u>trapezoid</u>: a quadrilateral with one and only one pair of parallel sides. In a parallelogram, <u>both</u> pairs of opposite sides must be parallel.
2. Draw two trapezoids as follows:





   Elicit that in ABCD, $\overline{AB}$ is parallel to $\overline{CD}$ but in A'B'C'D', side $\overline{A'D'}$ is parallel to $\overline{B'C'}$.
   Define: The parallel sides of a trapezoid are called its <u>bases</u>. The nonparallel sides are called <u>legs</u>.
   Ask, "What will be true about ∡A and ∡D?" (They are

supplementary since interior angles on the same side of a transversal that intersects two parallel lines are supplementary.) Challenge students to name other pairs of supplementary angles (∡B and ∡C, ∡A' and ∡B', ∡D' and ∡C').

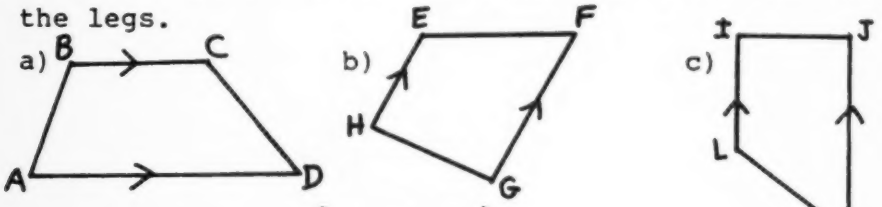
2. Ask, "Is it possible for a trapezoid to have a pair of opposite sides which are congruent?" (Yes, but only if they are the nonparallel sides. If the parallel sides were also congruent, it would force the figure to be a parallelogram.) Challenge the students to sketch a trapezoid with one pair of opposite sides congruent. Elicit the definition: A trapezoid with two congruent legs is called an <u>isosceles trapezoid</u>.

3. Give the students several well-drawn isosceles trapezoids. Ask them to measure the angles. Elicit <u>Property 1</u>: The base angles of an isosceles trapezoid are congruent. (<u>Note</u>: Each trapezoid has two pairs of base angles.)

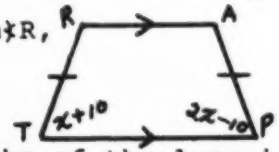
4. Have the students draw and measure the two diagonals. Elicit <u>Property 2</u>: The diagonals of an isosceles trapezoid are congruent.

APPLICATIONS:

1. In each figure indicate which are the bases and which are the legs.

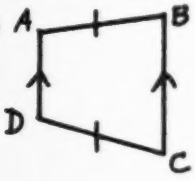


2. Find x, m∡T, m∡R, m∡A, m∡P.



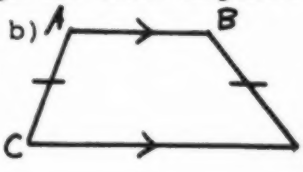
3. Find the lengths of the legs in each figure.

a)



AB = 2x + 15
DC = 4x - 5

b)



AB = 6y
DC = 2y + 50

4. In isosceles trapezoid WXYZ, $\overline{WX} \| \overline{YZ}$. If the measure of $\angle W = 2y - 9$ and the measure of $\angle Z = y + 14$ find y, $m\angle Z$, $m\angle Y$, $m\angle X$, $m\angle W$.

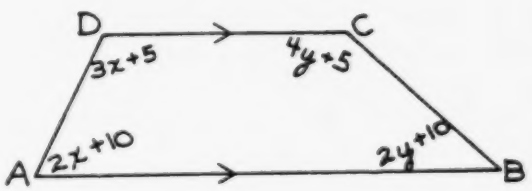
5. In trapezoid LMNO, $\overline{LM} \| \overline{NO}$. If $\overline{LN}$ and $\overline{MO}$ are drawn and LN = 4a - 11 and MO = 2a + 5, find LN.

SUMMARY:

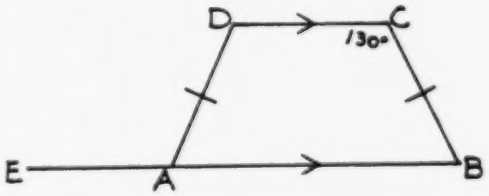
1. How does a trapezoid differ from a parallelogram?
2. What is an isosceles trapezoid?
3. What are the special properties of an isosceles trapezoid?
4. In the diagram of the set of quadrilaterals made in yesterday's lesson, where would you place the set of trapezoids? ...the set of isosceles trapezoids?

HOMEWORK:

1. One base angle of an isosceles trapezoid measures 63°. How large must each of the other 3 angles be?
2. For the trapezoid at the right, find the measures of the following:
   a) $\angle A$ c) $\angle D$
   b) $\angle D$ d) $\angle C$



3. For the trapezoid at the right, find the measures of the following:
   a) $\angle B$ c) $\angle D$
   b) $\angle A$ d) $\angle EAD$

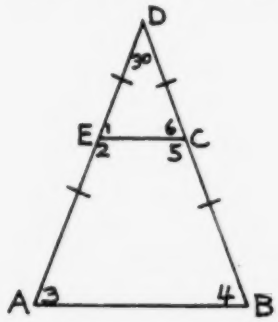


4. Is it possible for a trapezoid not to be completely in a plane? Explain.
5. If the two legs of a trapezoid are congruent, then we learned that:
   a) The base angles are congruent. State a converse. Do you think this converse is true?

b) The diagonals are congruent. State a converse. Do you think this converse is true?

6. If the opposite sides of a quadrilateral are congruent, must it be a /P/? Explain or show with toothpicks.

7. Find the measures of all six angles: 1,2,3,4,5,6. What seems to be true about $\overline{EC}$ and $\overline{AB}$? What kind of figure is ABCE?

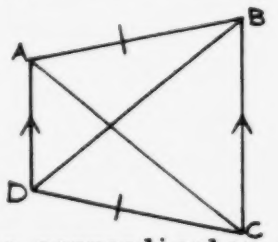


8. For the figure at the right we have

   AC = 7a + 23

   BD = 5a + 111

   Find AC and BD.



9. Draw two line segments that are perpendicular to and bisect each other. Connect the four endpoints to form a quadrilateral. What kind of quadrilateral does it seem to be? Check by measuring.

10. If all four sides of a quadrilateral are congruent, must it be a rhombus? Explain or show by using toothpicks.

Lesson 14 - Algebra and Geometry

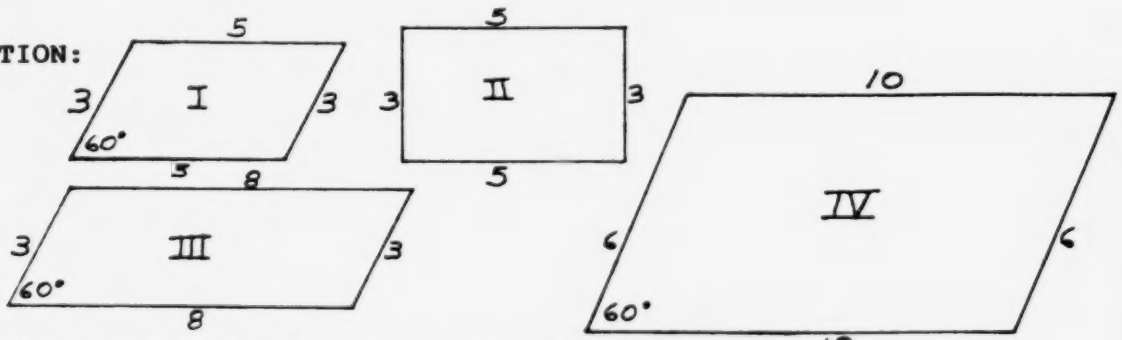
AIM: What are similar polygons?

PERFORMANCE OBJECTIVES: The student will be able to...

1. Define polygon.
2. Define similar polygons.
3. Define ratio and proportion.
4. Find the lengths of corresponding sides of similar figures.

VOCABULARY: polygon, similar polygons, ratio, proportion

MOTIVATION:



Which quadrilaterals seem most alike in shape?

DEVELOPMENT:

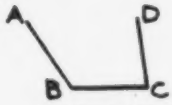
1. Discuss the motivation. Students should observe that all four figures are parallelograms; figures I, III and IV have the same angles; figures I and II have equal perimeters; figure II is a rectangle.
2. Students will probably suggest that figures I and IV are most alike. "What features of I and IV lead you to say they have the same shape?" Elicit that the angles of one are congruent to "corresponding" angles of the other, and the sides of one are are twice as long as the "corresponding" sides of the other. When the sides of figure IV are divided by the corresponding sides of figure I, the quotient (or ratio) is always equal to 2:1. A statement that two quotients are equal is called a

proportion. This proportion may be written $\frac{8}{4} = \frac{2}{1}$ or 8:4::2:1.

3. Our discussion of figures with the same shape is not limited to quadrilaterals.

   Define: Polygon - a simple closed shape made up of line segments.

   Note:

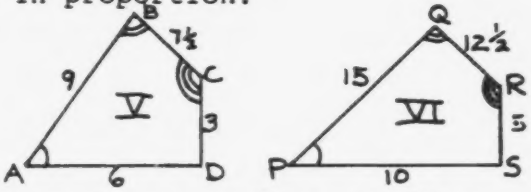


Not Closed

Not Simple

   Polygons are classified by number of sides, e.g., triangle, quadrilateral, pentagon, hexagon, etc.

4. "Similar" (like, opposite and positive) has a special meaning in mathematics. Define: Similar polygons are polygons whose "corresponding" angles are congruent and "corresponding" sides are in proportion.



   Here we say ABCD $\sim$ PQRS.

   Since $\frac{AB}{PQ} = \frac{BC}{QR} = \frac{CD}{RS} = \frac{DA}{SR}$ and $\measuredangle A \cong \measuredangle P$, $\measuredangle B \cong \measuredangle Q$, $\measuredangle C \cong \measuredangle R$, $\measuredangle D \cong \measuredangle S$.

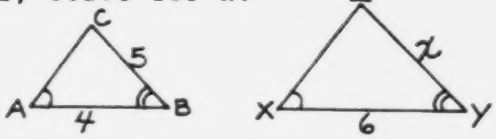
   The ratio of any pair of corresponding sides in similar polygons is called the <u>ratio of similitude</u>. Elicit that, in this case, the ratio of similitude of Polygon V to Polygon VI is 3:5.

APPLICATIONS:

1. Is the following a true proportion?
   a) $\frac{3}{7} = \frac{9}{21}$ (yes)
   b) $\frac{4}{6} = \frac{8}{10}$ (no)

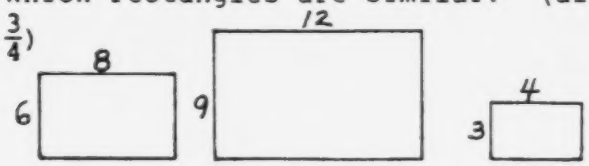
2. Solve for x in the true proportion $\frac{x}{5} = \frac{9}{15}$. (x = 3)

3. If $\triangle ABC \sim \triangle XYZ$, solve for x.



($x = 7\frac{1}{2}$)

4. Which rectangles are similar? (all are, since $\frac{6}{8} = \frac{9}{12} = \frac{3}{4}$)



I II III

5. A line segment 20" long is divided into segments which are in ratio 2:3. Find the length of each segment. (either $2x + 3x = 20$; $x = 4$; $2x = 8$, $3x = 12$ or $\frac{x}{20-x} = \frac{2}{3}$; $3x = 40 - 2x$, etc.)

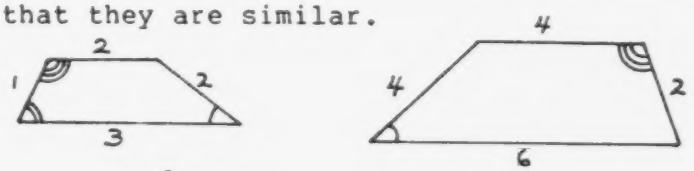
SUMMARY:

1. What are similar figures?
2. Are all rectangles similar?
3. Are all squares similar?
4. A photo of the Statue of Liberty is 3" x 5" (5" is the length). If we enlarge the photo so that it is 12" wide, how long will it be?

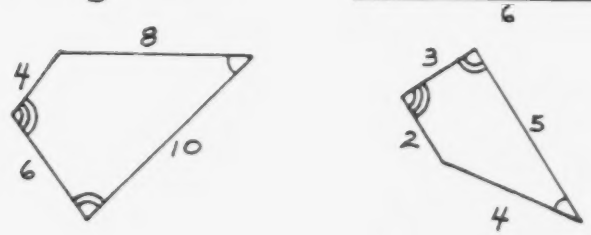
HOMEWORK:

1. Label the following quadrilaterals ABCD and A'B'C'D' to show that they are similar.

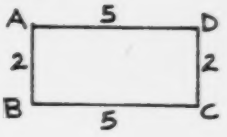
a)



b)

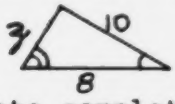


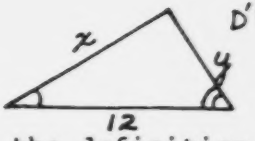
2. If rectangles ABCD and A'B'C'D' are similar, compute the missing lengths.



3. $\triangle ABC \sim \triangle DEF$

Find x, y, z.





4. State completely the definition of similar polygons.
"Two polygons are similar if... "

5. If $\triangle ABC \sim \triangle A'B'C'$, what six equalities must be true from the definition of similarity?

6. If the measures of the angles of a triangle are in the ratio of 2:3:5, how many degrees must the measure of each of the three angles have?

7. The perimeter of a quadrilateral is 100m. If the sides are in the ratio of 1:2:4:3, how long are the four sides?

Lesson 15 - Algebra and Geometry

AIM: What are some applications of similarity?

PERFORMANCE OBJECTIVES: The student will be able to...

Solve problems involving similarity of triangles.

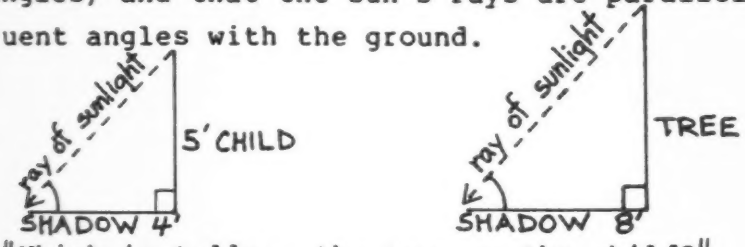
VOCABULARY: no new terms

MOTIVATION:

At the time a 5' child casts a 4' shadow, a maple tree casts an 8' shadow. How tall is the tree?

DEVELOPMENT:

1. Elicit that a diagram would be a useful tool in understanding the situation. It is reasonable to assume the tree and the child are standing vertically (creating 90° angles) and that the sun's rays are parallel forming congruent angles with the ground.



Ask: "Which is taller, the tree or the child?"
Students may suggest that the tree is twice as tall, because its shadow is twice as long.
Ask: "What assumption is being made about the triangles in the diagram?" (They are similar; lengths of their corresponding sides are in proportion.)

2. "Do we have the right to assume that if two angles of one triangle are congruent to two angles of another triangle, then the triangles are similar?
What investigation could we conduct to test this conjecture?" Elicit: draw two triangles, each with the same two angles; then measure the three sides of each triangle to determine if corresponding sides are in proportion. (It can be deduced that the third pair of angles must be congruent from theorem established

earlier.)

Draw a triangle on the board with one side 8" between angles with measures suggested by the class. Students should measure the other two sides. Ask students to draw triangles in their notebooks, with a side 4" long included between the same two angles. Are the triangles similar? (Repeat for various measures.)

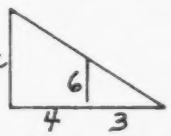
We may generalize that if two angles of one triangle are congruent to two angles of another, then the triangles are similar.

Since, in $\triangle ABC$ and $\triangle DEF$, we have $\measuredangle A \cong \measuredangle D$ and $\measuredangle B \cong \measuredangle E$, the triangles are $\sim$, and $\frac{x}{5} = \frac{8}{4}$ resulting in $x = 10$. (The tree is 10 feet tall.)

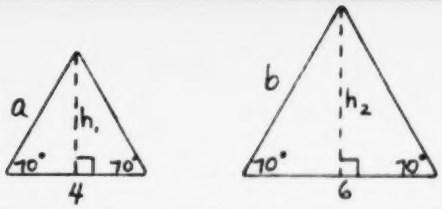
APPLICATIONS:

1. A pole casts a 9' shadow at the same time a yardstick casts a 2' shadow. Find the height of the pole. ($\frac{x}{3} = \frac{9}{2}$, $x = 13\frac{1}{2}'$)
2. A brother and sister cast 3' and 5' shadows respectively. If the sister is 60" tall, how tall is the brother? (36")
3. A 6' man stands within the shadow of a building as far from the building as he can. If he is 4' from the building and 3' from the end of the shadow, how tall is the building?

   $\frac{x}{6} = \frac{4+3}{3} \longrightarrow x = 14'$

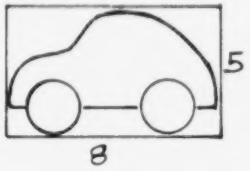
4. Two tents, each in the shape of an isosceles triangle, have base angles measuring 70°. If the smaller has a 4' base and the larger a 6' base, what is the ratio of the length of a "leg" in the larger tent to that of the smaller tent? What is the ratio of the heights of the tents? (Note that the altitude, one leg and its projection on the base form a triangle and a triangle for one tent is similar to its corresponding triangle for the other tent.)

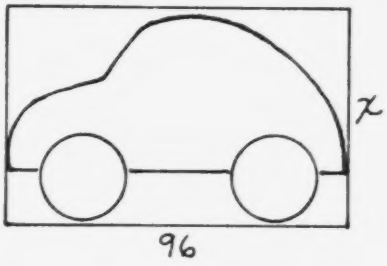


$\frac{b}{a} = \frac{6}{4}$, but $\frac{h_2}{h_1} = \frac{b}{a}$

so $\frac{h_2}{h_1} = \frac{6}{4}$ also

5. A model car is constructed "similar" to the original. If the model is 5" high and 8" long, and the original is 96" long, how high is the original?





$\frac{x}{5} = \frac{96}{8}$, $x = 60$

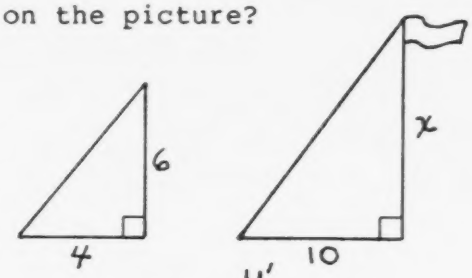
SUMMARY:

Ask the students to write an explanation of how a girl can use her height and the length of her shadow to determine the height of certain objects.

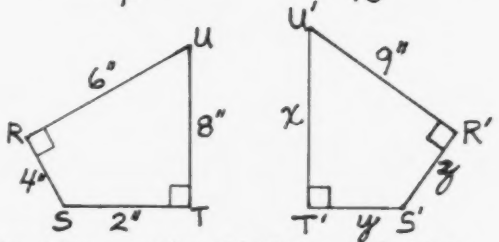
HOMEWORK:

1. A faithful enlargement of a picture, by a factor of 3, shows a monument that measures 18" on the enlargement. What does the monument measure on the picture?

2. A flagpole casts a 10' shadow at the moment a man 6' tall casts a 4' shadow. How high is the flagpole?



3. Quadrilaterals RSTU and R'S'T'U' are similar. Find the missing segments as well as their perimeters.



4. The sides of a triangle are in the ratio 3:4:5. If the perimeter of this triangle is 36 inches, how long is each side. Check to see if the sum of the squares of the lengths of the two smaller sides is the same as the square of the length of the largest side.

5. The angles of a quadrilateral are in the ratio 2:3:3:4. How large is each of the angles measured in degrees?

Lesson 16 - Algebra and Geometry

AIM: What are congruent polygons?

PERFORMANCE OBJECTIVES: The student will be able to...

1. Define congruent polygons.
2. Recognize pairs of congruent triangles by SAS, ASA and SSS.

VOCABULARY: congruent polygons

MOTIVATION:

Tape to the board a preconstructed 30°-60°-90° triangle with hypotenuse = 11" (do not label measures). Ask class, "If I told you to construct a triangle identical to this one, how would you proceed?"

DEVELOPMENT:

1. Students will probably ask for the measure of each angle and the length of each side. Offer three paper strips or sticks which are congruent to the sides of the given triangle. The students will see that they can be assembled in only one way. Explain that we have accomplished the objective using only 3 of the 6 parts of the triangle.
2. Define congruent ($\cong$) polygons as polygons having "corresponding" sides and "corresponding" angles congruent. Elicit that $\cong$ polygons are also $\sim$, but $\sim$ polygons are not always $\cong$. More generally, two polygons are congruent if they can be named ABC and A'B'C' so that $\measuredangle A = \measuredangle A'$, $\measuredangle B = \measuredangle B'$... and $AB = A'B'$, $BC = B'C'$... When are similar polygons congruent? (When the ratio of similitude is 1:1.)
3. Ask: "Will any three pairs of congruent parts suffice to guarantee congruent triangles?" Suppose only three angles were known. (Show a smaller or larger cutout triangle which is similar but not congruent to the original one.)

4. Divide the class into two groups. Ask one group to draw a line segment 11" long, a 60° angle at the end of the segment, and mark off $5\frac{1}{2}$" along the second ray of the angle. Make a triangle by connecting the ends of the segments. Have the other group draw the 11" segment with a 30° angle at one end and a 60° angle at the other. Extend the rays of each angle until they intersect.

5. Compare students' triangles. Cut several out from each group and show by measuring and/or "overlapping" that the triangles are $\cong$.

   Ask, "Which combination of measures produced triangles which were congruent to the given triangle?" (3)

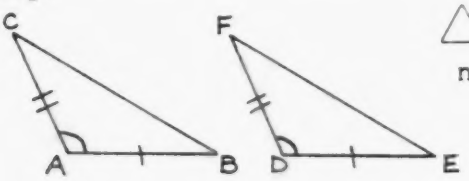
   List: Two triangles are congruent if

   a) Two sides and the included angle of one are $\cong$ to corresponding parts of the other (SAS).

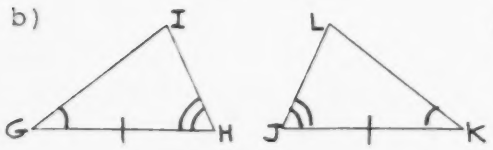
   b) Two angles and the included side of one are $\cong$ to corresponding parts of the other (ASA).

   c) Three sides of one triangle are $\cong$ to corresponding sides of the other (SSS).

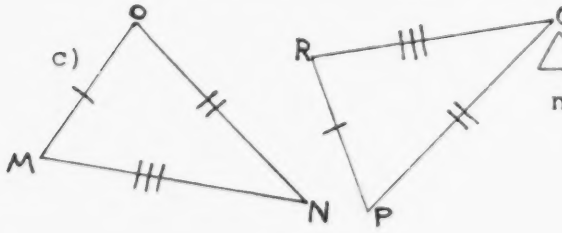
   Therefore, given the facts marked off in each diagram, we can say:

a) 

△ABC $\cong$ △DEF (SAS)
name corresponding parts

b) 

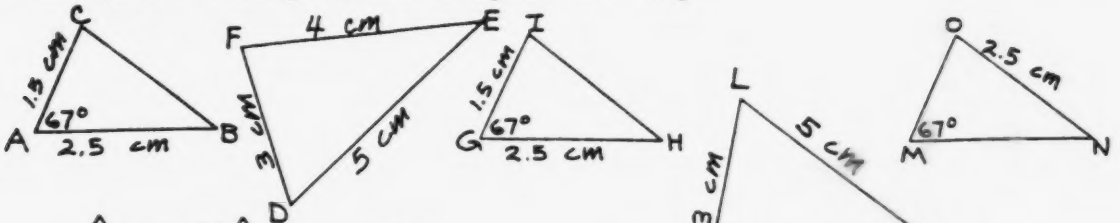
△GHI $\cong$ △KJL (ASA)
name corresponding parts

c) 

△MNO $\cong$ △RQP (SSS)
name corresponding parts

APPLICATIONS:

1. Which triangles are congruent? Why?



$\triangle ABC \cong \triangle GHI$ (SAS)

$\triangle DEF \cong \triangle LJK$ (SSS)

Note: $\triangle MNO$ is not $\cong \triangle ABC$ since 67° is not included between the congruent sides.

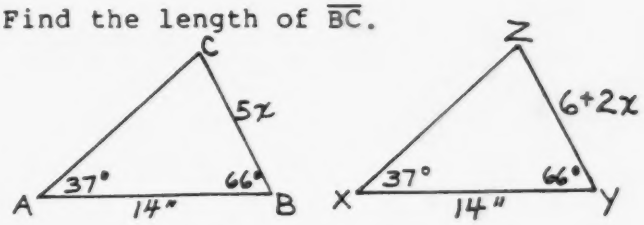
2. Is $\triangle ABC \cong \triangle XYZ$? Why? Find the length of $\overline{BC}$.

($\cong$ by ASA, so

$5x = 6 + 2x$

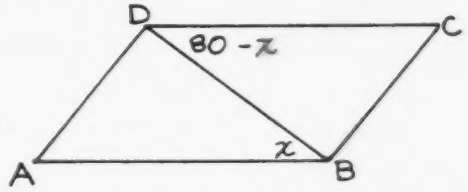
$3x = 6$, $x = 2$, $5x = 10$

$BC = 10$)



3. a) If ABCD is a parallelogram is $\triangle ABD \cong \triangle CDB$?

(Elicit $\overline{AB} \cong \overline{CD}$, $\overline{AD} \cong \overline{CB}$ and $\overline{BD} \cong \overline{DB}$, $\triangle \cong \triangle$ SSS)

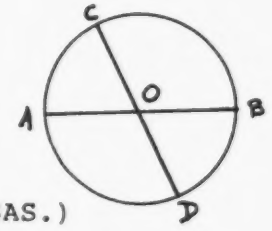


b) Find $\measuredangle ABD$. ($x = 80 - x$, $2x = 80$, $x = 40$ $\measuredangle ABD = 40°$)

Elicit: The diagonal of any parallelogram divides it into two congruent triangles.

4. If O is the center of the circle with diameters $\overline{AB}$ and $\overline{CD}$, what triangles can be shown to be $\cong$?

(Since all radii of circle O must be $\cong$ and since vertical angles are $\cong$, $\triangle AOC \cong \triangle BOD$ and $\triangle AOD \cong \triangle COB$, both by SAS.)



SUMMARY:

1. What must be demonstrated to establish that two triangles are congruent?

2. Why is it useful to be able to prove that triangles are congruent? (Since only <u>three</u> congruent pairs need be shown to prove that figures are congruent, another three pairs of congruent parts are a happy consequence. Congruence can be useful fo find unknown measures.)

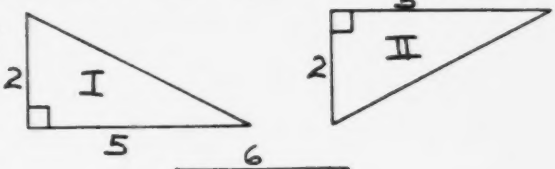
3. How many/which pairs of parts would you have to show congruent to prove that two quadrilaterals are congruent?

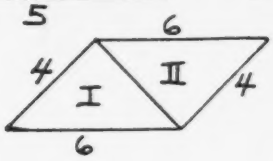
HOMEWORK:

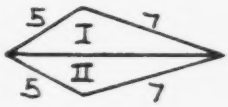
1. a) According to the <u>definition</u>, what six equalities must hold for $\triangle ABC \cong \triangle A'B'C'$?

   b) What three equalities are sufficient for $\triangle ABC \cong \triangle A'B'C'$? (We have learned of three sets of three equalities. Give all three.)

2. In each case below, the triangles are congruent. Explain why.

   a) 

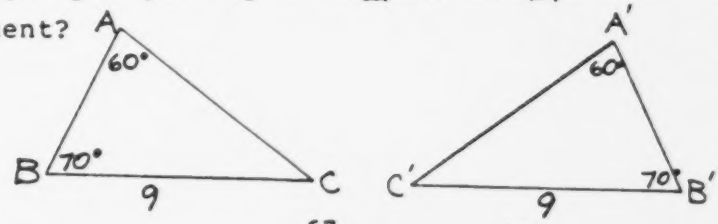
   b) 

   c) 

   d)

3. If quadrilaterals XYZW and X'Y'Z'W' are congruent, what eight equalities must hold according to the definition of congruence?

4. How might you justify that $\triangle ABC$ and $\triangle A'B'C'$ are congruent?



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5. Three numbers are in the ratio 3:4:5. If the largest number is 8 more than the smallest, what must the three numbers be?

6. Suppose that for $\triangle ABC$ and $\triangle A'B'C'$ we have the following three equalities: $\measuredangle A = \measuredangle A'$, $\measuredangle B = \measuredangle B'$ and $BC = B'C'$. Must they be congruent? If not, show by example.

7. The diagonal of any parallelogram divides it into two congruent triangles.
   a) Write this conditional statement in if-then form.
   b) State its converse.
   c) Is the converse a true statement? Prove or show a counterexample.

Lesson 17 - Algebra and Geometry

AIM: How do we construct a triangle congruent to a given triangle?
How do we bisect a given segment?

PERFORMANCE OBJECTIVES: The student will be able to...

1. Use a compass.
2. Construct a line segment $\cong$ to a given segment.
3. Construct a triangle $\cong$ to a given triangle.
4. Bisect a line segment.

VOCABULARY: bisect

MOTIVATION:

Draw a triangle on the board. Ask: "How can we make a second triangle which is congruent to this one?"

DEVELOPMENT:

1. Discuss the motivation problem. Students will suggest measuring parts and making a second traingle using SAS, ASA or SSS. Ask: "How could this be done if you had no ruler or protractor?" Students may suggest tracing the figure or using string or paper strips to measure the lengths of the sides.
2. Introduce compass and straight edge (no markings) as tools for <u>constructions</u> in geometry. The compass may be used to draw a circle or an arc or to "copy" a segment. We will <u>construct</u> a triangle congruent to the one which is given relying on the SSS postulate. First we will learn how to construct a segment congruent to a given segment. After achieving this simple objective, we will attempt the larger task of which it is a part.

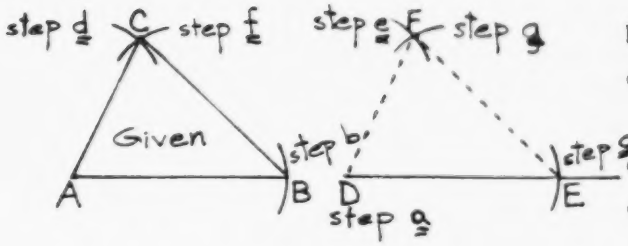
Given

Place the compass point at A and pencil point at B; this measures $\overline{AB}$.

Place compass point at C, maintain same compass setting and mark off point D.

Now $\overline{AB} \cong \overline{CD}$.

3. Now we are ready to construct a triangle congruent to a given triangle.



STEPS:

a) Draw a line and select point D on it.
b) Measure $\overline{AB}$ with a compass.
c) Mark off $\overline{DE} \cong \overline{AB}$. (Locate point E.)
d) Measure $\overline{AC}$ with compass.
e) Mark off an arc congruent to $\overline{AC}$ with its center at D. (With same setting, place point at D and make an arc.)
f) Measure $\overline{BC}$ with compass.
g) Mark off an arc congruent to $\overline{BC}$ with its center at B.
h) Label the point where the arcs intercept F. (This is the third vertex of the triangle.)
i) Using straightedge, draw $\overline{DF}$ and $\overline{EF}$.

Now $\triangle DEF \cong \triangle ABC$.

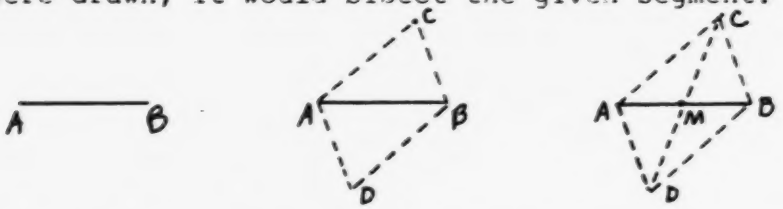
4. Practice this construction several times.

5. Pose a new challenge: Draw a segment, and ask, "There is a point on this segment (called the midpoint) which divides the segment into two exactly congruent parts. How would you locate this point without using a ruler?" Some ingenious student may suggest tracing the segment and folding the paper so that the endpoints coincide.

Another approach is to lay a piece of string along the segment and fold the string in half.

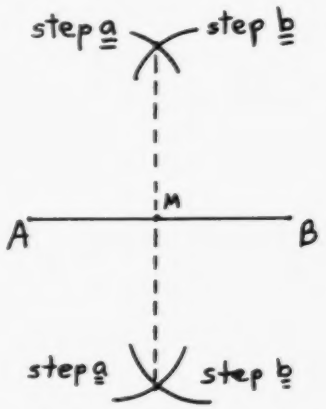
Ask the students to recall a fact discovered previously (when studying quadrilaterals) about segments which bisected each other: the diagonals of any parallelogram bisect each other. This fact may be used as the basis of a method for locating the midpoint of a given segment (bisecting a given segment).

Imagine that the given segment is one of the diagonals of a parallelogram; its endpoints are opposite vertices of the parallelogram. If the other two vertices of the parallelogram could be located, and the second diagonal were drawn, it would bisect the given segment.



Since we have learned how to copy a segment, we can create the desired paralellogram. The easiest type of parallelogram in which to label the given segment as a diagonal would be a rhombus: we could use the same setting for the lengths of all four sides.

6. To bisect a line segment (divide into two = segments).
   a) Point of compass at A, draw arc above and below $\overline{AB}$. (Use any radius which is more than half AB.)
   b) Point of compass at B, repeat process with same compass opening.
   c) Label points of intersection of arcs C and D. Connect $\overline{CD}$. Now $\overline{CD}$ bisects $\overline{AD}$.



   d) The point of intersection of $\overline{AB}$ and $\overline{CD}$ is M, which is the midpoint of $\overline{AB}$. $\overline{AM} \cong \overline{MB}$.

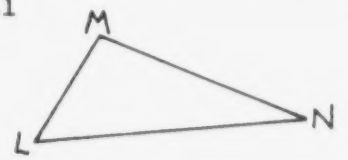
7. Practice this construction.

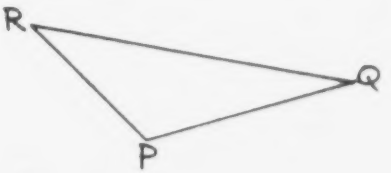
APPLICATIONS:

Have students perform construction (on prepared rexo sheet):

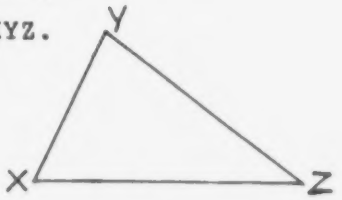
1. a) Construct a segment $\cong \overline{AB}$.
   b) Construct a segment $\cong \overline{XY}$.
   c) Construct a segment equal in length to AB + XY.

2. a) Construct a triangle $\cong \triangle LMN$.
   b) Construct a triangle $\cong \triangle PQR$.





3. a) Bisect $\overline{AB}$.
   b) Bisect each side of $\triangle XYZ$.



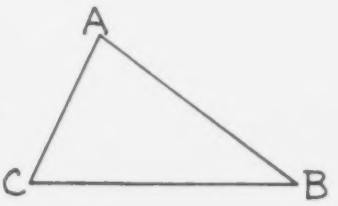
SUMMARY:

1. What tools are allowed to execute the direction "construct"?
2. Name three functions of a compass.
3. What constructions have we learned to perform today? (Describe the procedure for each.)

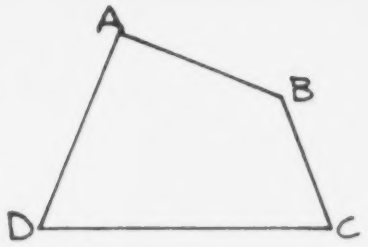
HOMEWORK:

1. By two bisections, find a point X on $\overline{AB}$ so that $AX = \frac{1}{4}AB$.

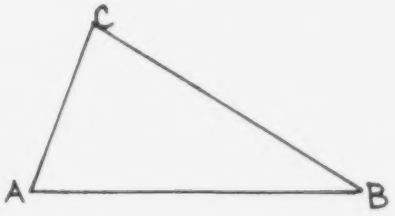
2. Find the midpoints of $\overline{AB}$ and $\overline{AC}$. Connect them. How does this line segment compare with $\overline{BC}$?



3. Find the midpoints of all four sides of quadrilateral ABCD and connect them in order. What kind of figure seems to result?



4. For $\triangle ABC$, call the midpoints of $\overline{AB}$, $\overline{BC}$ and $\overline{CA}$, R, S and T respectively. Draw $\overline{AR}$, $\overline{BS}$ and $\overline{CT}$. What seems to happen?

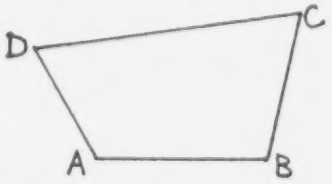


5. Draw $\triangle ABC$ and locate by construction the midpoints of its sides (points R,S,T). Draw $\triangle RST$. What is the relationship between $\triangle RST$ and $\triangle ABC$? Between $\triangle RST$ and any other triangle?

6. Divide $\overline{AB}$ in the ratio 3:5.

7. Construct a triangle with sides of the lengths 2", 3" and 4".

8. Construct quadrilateral A'B'C'D' = quadrilateral ABCD on $\overline{A'B'}$.



9. a) Four numbers are in the ratio 2:4:3:7. If the sum of the largest and smallest is 144, how large are the four numbers?

   b) On the given segment, locate points A, B, C so that the segment is divided into parts whose lengths are in the ratio 2:4:3:7.

Lesson 18 - Algebra and Geometry

AIM: How do we bisect an angle?
How do we construct an angle congruent to one which is given?
How do we construct perpendicular lines?

PERFORMANCE OBJECTIVES: The student will be able to...

1. Bisect an angle.
2. Reproduce an angle.
3. Construct a line perpendicular to a given line through a point not on the line.

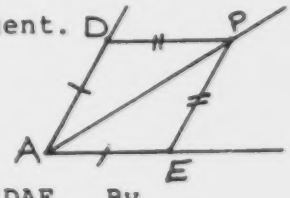
VOCABULARY: perpendicular

MOTIVATION:

1. Use a protractor to draw an angle ($\measuredangle CAB$) measuring 70°.
2. Without the protractor, try to draw a line through A so that $\measuredangle CAB$ is divided into two $\cong$ angles.
3. Measure with the protractor to check how well the line you drew bisected the angle.

DEVELOPMENT:

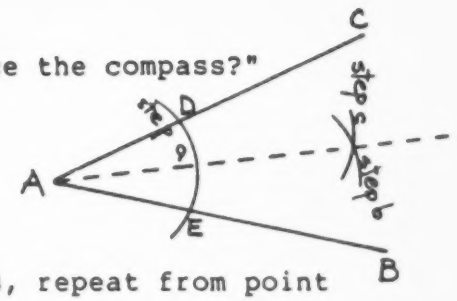
1. We can use a compass to bisect an angle. Elicit that the class has already learned a way to prove that two angles are congruent: if they are corresponding parts in congruent triangles, then they are congruent.
Draw the figure:
Ask: "Are the triangles congruent? Why?"



Observe that $\measuredangle DAP \cong \measuredangle EAP$ and $\overline{AP}$ bisects $\measuredangle DAE$. By imitating this situation, we can design the procedure for the new construction.

Given ∠CAB, ask, "Where should we place the compass?"

a) Place compass point at A, draw an arc through $\overline{AC}$ and $\overline{AB}$, intersecting at points D, E.



b) (Try to elicit) place compass point at E, draw arc within ∠CAB, repeat from point D with the same compass opening.

c) (Try to elicit) connect the intersection of these arcs to point A. Now $\overline{PA}$ bisects ∠CAB.

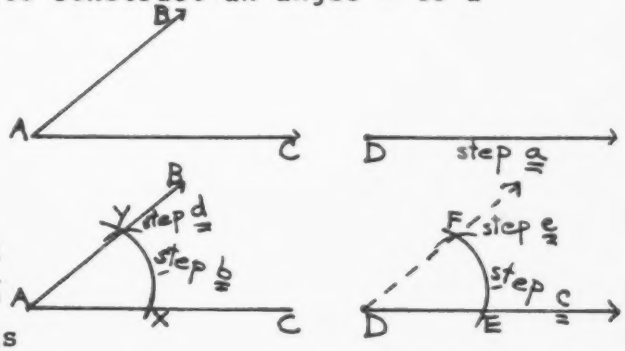
Practice this construction (see applications) before continuing.

2. Ask: What type of angles result when a straight angle is bisected? What is the relationship between the angle bisector and the original line? (They are perpendicular.)

3. Yesterday we learned to "copy" a segment and to "copy" a triangle. Now we will develop a procedure which may be used to reproduce an angle (to construct an angle ≅ to a given angle). Given: ∠ABC

a) Locate a point to be the vertex of the new angle and locate its first side.

b) Place compass point at A, draw arc through $\overline{AB}$ and $\overline{AC}$, locating points X, Y.



c) With the same compass setting, place the point at D and draw an arc, locating E.

d) Use compass point and pencil to "measure" $\overline{XY}$ (place compass point at X and pencil point at Y).

e) With the same compass setting, place the point at E and draw an arc through the existing arc. Label the intersection F.

f) Draw $\overrightarrow{DF}$.

Now ∠FDE ≅ ∠CAB. "Why?" ($\overline{DE} \cong \overline{AX}$, $\overline{DF} \cong \overline{AY}$, $\overline{EF} \cong \overline{XY}$, etc.)

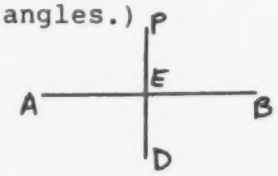
Practice: (see applications).

4. Draw this figure and say:

"Suppose we wanted to construct a line through P which is perpendicular to segment AB. Is this situation the same as the one we considered earlier, when bisecting a straight angle?" (No. In that case, point P was <u>on</u> the given line - the vertex of the straight angle. Now point P is not on the given line.)

Let us develop a procedure that we can use to construct a line perpendicular to a given line from a point not on the line. (If necessary, define perpendicular lines as two lines which intersect at right angles.)

$\overline{AB}$ $\overline{PD}$ if ∡AEP, ∡PEB, ∡BED, ∡DEA are right angles.



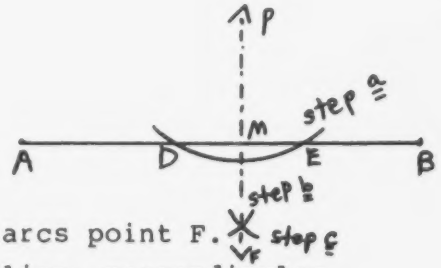
a) Place compass point at P, draw an arc large enough to intersect $\overline{AB}$ in 2 points (label these D, E).

b) Draw an arc below $\overline{AB}$, with the compass point at D.

c) With the same compass setting, repeat step b) at point E.

d) Label the intersection of these arcs point F.

e) Draw $\overleftrightarrow{PF}$. This is the desired line, perpendicular to the given segment $\overline{AB}$.



(Note: M is the midpoint of $\overline{DE}$ but is not necessarily the midpoint of $\overline{AB}$.) Ask, "Why does this procedure work?"

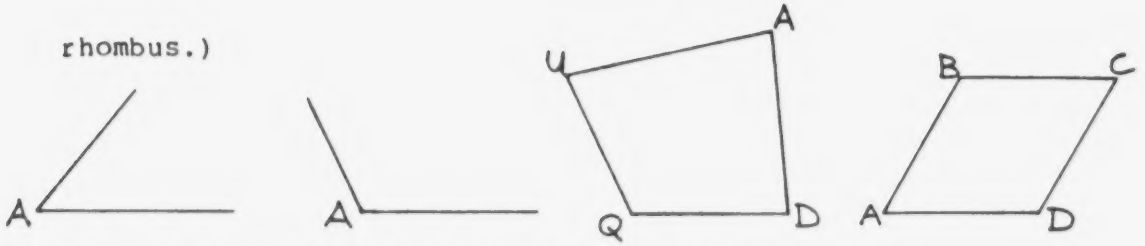
(Honor assignment, perhaps as homework: △DPF ≅ △EPF by SSS, so ∡DPF ≅ ∡EPF; △DPM ~ △EPM by SAS and the corresponding angles with vertices at M must be congruent.)

APPLICATIONS:

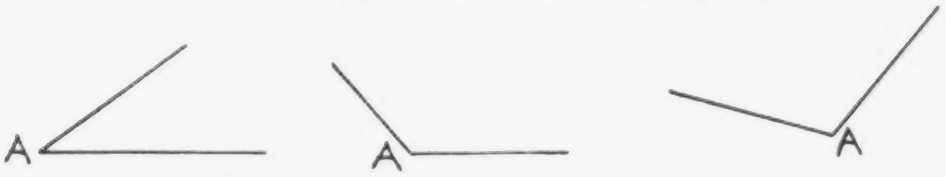
Provide class examples on a rexo sheet.

1. Bisect ∡A in each diagram. (Note: Prepare ABCD as a

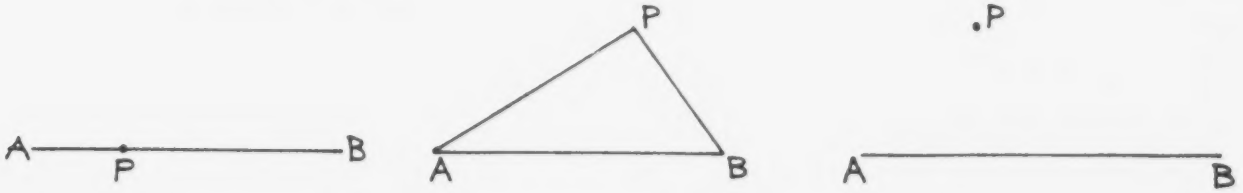
rhombus.)



2. Have students reproduce ∡A in each case.



3. Have students construct a line through P⊥$\overline{AB}$ (note in the second case, this line is the altitude of the triangle to $\overline{AB}$.)



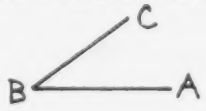
SUMMARY:

1. Describe how you would proceed to bisect an angle; to copy an angle; to construct a right angle.
2. How would you construct a line through P⊥$\overline{AB}$:
   a) if P were on $\overline{AB}$? b) if P were not on $\overline{AB}$?

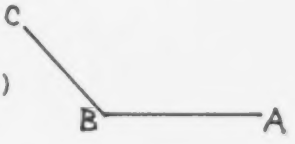
HOMEWORK:

1. In each case bisect ∡ABC.

a)



b)



c)

2. Construct a line through P⊥$\overline{AC}$. (You have constructed four 90° angles.)

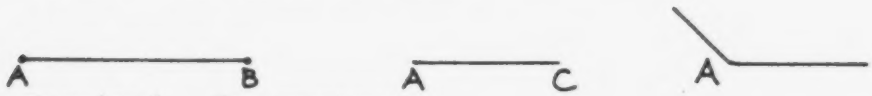
3. Construct a 45° angle without a compass.

4. Construct a 135° angle without a compass.

5. Given: The parts of $\triangle ABC$:

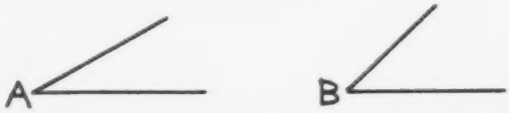
Required: To construct
$\triangle A'B'C' \cong \triangle ABC$ on line ℓ.

6. Given: The parts of $\triangle ABC$:



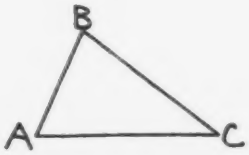
Required: To construct
$\triangle A'B'C' \cong \triangle ABC$ on line ℓ.

7. Given:

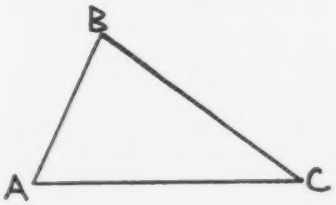


Required: To construct an angle that has the measure of the sum of $\angle A$ and $\angle B$.

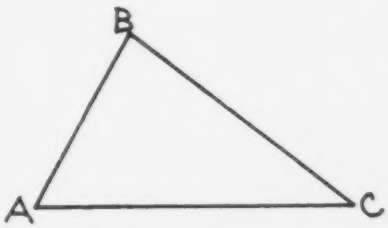
8. Construct an angle that has the measure of $\angle A$ + $m\angle B$ + $m\angle C$.



9. Construct the $\perp$ from each vertex to the opposite side of $\triangle ABC$. (Each is called an altitude.) What seems to happen? Experiment with other triangles.



10. Construct the angle bisector of each angle. What seems to happen? Experiment with other triangles.



11. Try to justify the procedure for constructing a perpendicular to a line from a point not in the line.

Lesson 19 - Algebra and Geometry

AIM: What is an isosceles triangle?

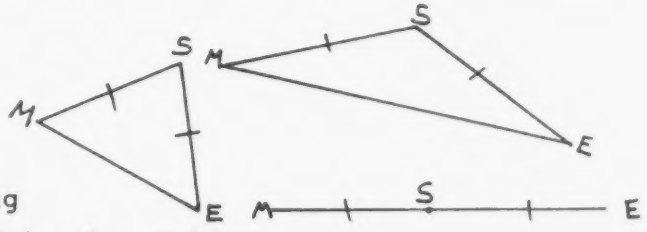
PERFORMANCE OBJECTIVES: The student will be able to...

1. Define an isosceles triangle.
2. Construct an isosceles triangle.
3. Identify and name the parts of an isosceles triangle.

VOCABULARY: isosceles triangle, vertex, legs, base angles, base

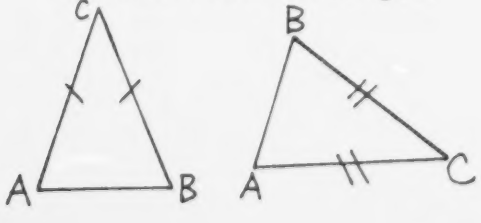
MOTIVATION:

Mary and her friend Eleanor live equally far away from the school they both attend. If Mary goes to visit Eleanor after school and later goes straight home, make a diagram to illustrate her travels from the time she left her house in the morning.



DEVELOPMENT:

1. Many different diagrams can be made, each meeting the conditions specified in the problem.
   Mary's path follows the sides of a triangle. (In all cases, points M and E lie on a circle whose center is at S. In the extreme case (longest path), $\overline{MSE}$ is a diameter. Otherwise, MS + SE > ME.)
   Elicit that two sides of this triangle are congruent.
   Recall the definition of an isosceles triangle as a triangle with two congruent sides.
2. We will construct an isosceles triangle using given sides, after reviewing the nomenclature for the parts of an isosceles triangle.

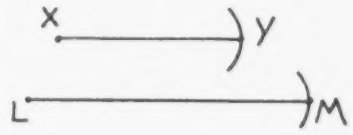


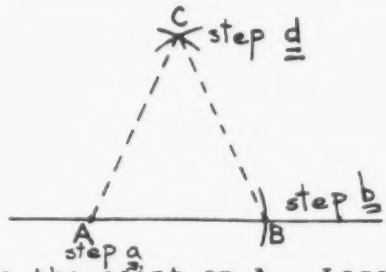
Legs: $\overline{AC}$, $\overline{BC}$
Base: $\overline{AB}$
Vertex angle: $\angle C$
Base angles: $\angle A$, $\angle B$

90

The congruent sides are LEGS, the third side is the BASE. The angles formed by each leg and the base (i.e., the angles opposite the legs) are BASE ANGLES. The angle formed by the two legs (i.e., the angle opposite the base) is the VERTEX ANGLE.

3. Construct an isosceles triangle with a base = $\overline{XY}$ and each leg = $\overline{LM}$.



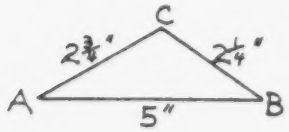


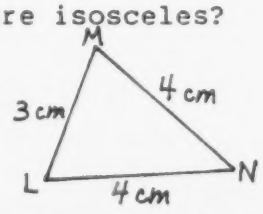
   a) Select point A (the first vertex) on a line which will contain the base.
   b) Set the compass opening equal to the length of $\overline{XY}$ and place the point on A. Locate B by drawing an arc through the line.
   c) Set the compass equal to the length of $\overline{LM}$.
   d) Draw an arc with the point at A and another at B.
   e) The intersection of these arcs is the vertex, C.
   g) Connect $\overline{AC}$ and $\overline{BC}$.

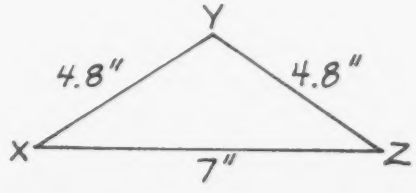
Triangle ABC is isosceles with $\overline{AC} \cong \overline{BC}$.

APPLICATIONS:

1. Which triangles are isosceles?

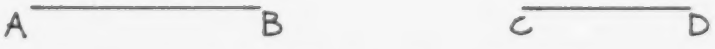






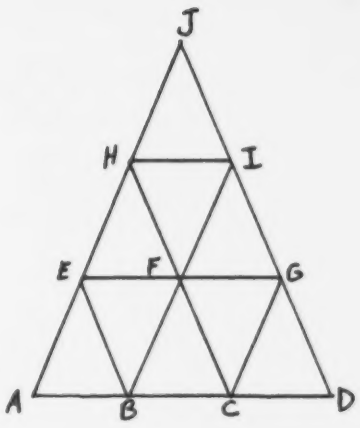
2. For each isosceles triangle in 1, identify: the legs, the base, the base angles and the vertex angle.

3. Construct an isosceles triangle with base $\cong \overline{AB}$ and leg $\cong \overline{CD}$.



4. Construct an equilateral triangle with sides each $\cong \overline{CD}$ (from exercise 3).

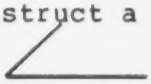
5. How many isosceles triangles can be found in this diagram? Note that JI = JH, HE = HF, IF = JC, etc.
(13 in total: △ABE, △BEF, △BCF, △CGF, △CDG, △EFH, △FIH, △FIG, △HIJ, △ACH, △BID, △EGJ, △ADJ)



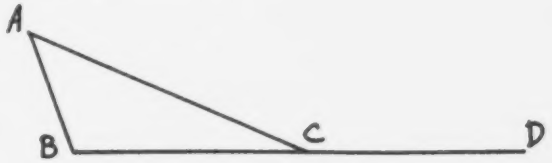
SUMMARY:

1. Is it possible for the base of an isosceles triangle to be the longest side? ...the shortest?
2. Is it possible for a vertex angle to be obtuse? ...a base angle...? (Explain.)
3. If a triangle has <u>three</u> congruent sides, is it isosceles? (Yes, an equilateral triangle is a special type of isosceles triangle, just as a square is a special type of rectangle.)

HOMEWORK:

1. Draw a triangle with sides of 2", 2" and 3". How many degrees are there in each of the three angles? What seems to be the case for the base angles of an isosceles triangle?
2. Construct a triangle with two angles having the same size as  . Measure all three sides. What seems to be the case if two angles of a triangle are congruent?
3. Construct a triangle with all three sides the same length as $\overline{AB}$.

Measure the three angles. What seems to be the case?
4. Construct an angle of 60°; of 30°.
5. Draw △ABC. Construct an angle inside ∡ACD with its vertex at C, one side on $\overline{CA}$ and congruent to ∡A.



Compare the size of ∡B with the rest of ∡ACD. What seems to be the relationship between the size of ∡ACD and the sum of the measures of angles A and B?
Experiment with other triangles. Can you make a generalization?

6. How many degrees are in each angle of a triangle if they are in the ratio of 2:5:5? Construct a triangle that meets this condition. What kind of triangle is it?

Lesson 20 - Algebra and Geometry

AIM: What are the properties of an isosceles triangle?

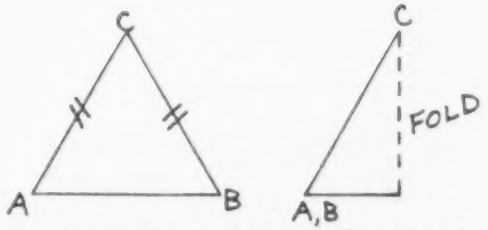
PERFORMANCE OBJECTIVES: The student will be able to...

1. State the base angles theorem.
2. State the converse of the base angles theorem.
3. Solve numeric and algebraic problems involving isosceles triangles.

VOCABULARY: no new terms

MOTIVATION:

Ask each student to construct a triangle ABC with $\overline{AC} \cong \overline{BC}$. Then ask students to fold the paper so that point B coincides with point A. What can the class conclude about angle A and angle B?

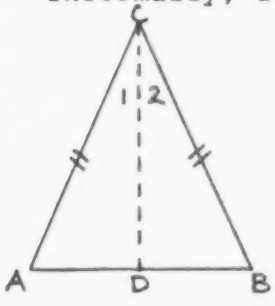


DEVELOPMENT:

1. Elicit the opinion that the base angles of an isosceles triangle are congruent. Renew concept of "theorem" (lesson 5).

   Theorem: If two sides of a triangle are congruent, then the angles opposite them are congruent.

   Informally, discuss the proof of the theorem.



We know $\overline{AC} \cong \overline{BC}$.

We can construct a line through C to bisect $\angle ACB$. (It would intersect $\overline{AB}$ at a point. Call this point D.)

$\angle 1 \cong \angle 2$

Also $\overline{CD} = \overline{CD}$

$\triangle ACD \cong \triangle BCD$ Why? (SAS)

$\angle A \cong \angle B$

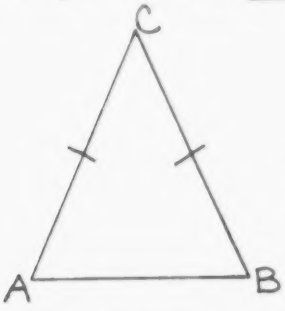
2. Ask students to state the converse of the theorem.
Converse: If two angles of a triangle are congruent, the sides opposite them are congruent.
Ask: Is this true? How would you prove it? (Again, begin with the angle bisector of the vertex angle.)

3. If, in isosceles $\triangle ABC$, $\overline{AC} \cong \overline{BC}$, the measure of $\angle A = 70°$, how could we find $m\angle B$, $m\angle C$? ($\angle B$ is a base angle, so it contains 70°. 40° remain for $\angle C$.)

APPLICATIONS:

1. If in isosceles triangle LMN, vertex angle N measures 80°, how can we find $\angle L$ and $\angle M$? ($80+x+x = 180$, $x = 50°$)

Using isosceles $\triangle ABC$, $\overline{AC} \cong \overline{BC}$:



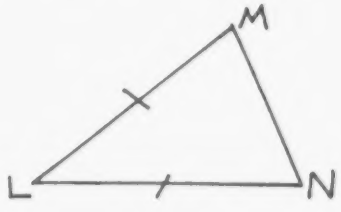
2. If $\angle A = 65$, find $m\angle B$ and $m\angle C$. (65°,50°)

3. Same if $\angle A = 55°$. (55°,70°)

4. If $m\angle C = 30°$, find $m\angle A$ and $m\angle B$. (each 75°)

5. If $m\angle A = 2x + 10$ and $m\angle B = 4x - 18$, find $m\angle A$. ($2x + 10 = 4x - 18$, $2x = 28$, $x = 14$, $m\angle A = 38°$)

6. In $\triangle LMN$, $\overline{LM} \cong \overline{LN}$.
If $m\angle M = 62°$, find $m\angle L$ and $m\angle N$.
(Here $m\angle N = 62°$, $m\angle L = 56°$)



7. In $\triangle PQR$, $m\angle P = 30°$, $m\angle Q = 30°$.
Find the length of $\overline{RP}$ if $RP = 8x$ and $RQ = 5x + 15$.
($8x = 5x + 15$, $3x = 15$, $x = 5$, $RP = 40$)

SUMMARY:

1. Ask class: What conclusions can we reach concerning $\triangle ABC$ if we know $\overline{CA} = \overline{CB}$? (Isosceles $\triangle$, $\angle A \cong \angle B$)

2. In $\triangle XYZ$ what can we conclude if we know $\angle X = \angle Y$? (Isosceles $\triangle$, $\overline{ZX} \cong \overline{ZY}$)

3. If the vertex angle of an isosceles triangle is congruent to the vertex angle of another isosceles triangle, will the triangles be similar? (yes) Explain.

4. If a base angle of an isosceles triangle is congruent to the base angle of another isosceles triangle, will the triangles be similar? (yes) Explain.

5. If one angle of an isosceles triangle is congruent to one angle of another isosceles triangle, will the triangles be similar? (Not necessarily: the angle might be a vertex angle in one triangle and a base angle in the other.)

HOMEWORK:

1. The vertex angle of an isosceles triangle measures 40°. How many degrees are there in the measure of each base angle?

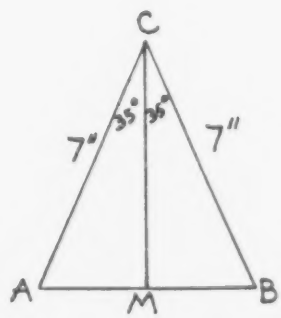
2. A base angle of an isosceles triangle measures 25°. How many degrees are there in the measure of the vertex angle?

3. Show that if all three angles of a triangle have the same measure, all three sides must have the same measure.

4. Which of the following triangles are isosceles?

	∠A	∠B	∠C
a)	45°	90°	___
b)	60°	___	60°
c)	56°	63°	___

5. Show that $\overline{AM} \cong \overline{BM}$ in the figure at the right.



Lesson 21 - Algebra and Geometry

AIM: What is meant by the area of a quadrilateral?

PERFORMANCE OBJECTIVES: The student will be able to...

1. Define area.
2. Find the area of a:
   a) rectangle
   b) parallelogram

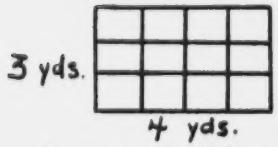
VOCABULARY: area

MOTIVATION:

How much would it cost to buy carpet for a bedroom 12' long by 9' wide if carpet costs $8 per square yard?

DEVELOPMENT:

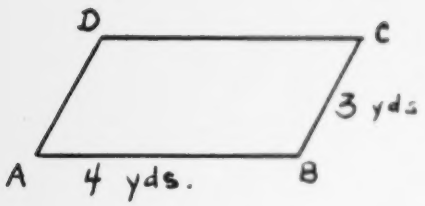
1. Ask: "What does area mean?" An informal definition might be the amount of room within a closed figure (in a plane or 2 dimensions). Just as the length of a line segment is measured by specific units (inch, cm, etc.), we use specific units to measure area. (Using your notebook as a unit of area, you could find the area of your desk top.) Common units of measure are square inch, square cm, square yard, etc. A "square inch" is the area of a square whose side is 1" long.
2. Discuss the solution to the "carpet problem". Elicit: 12' = 4 yds., 9' = 3 yds.; area = 12 sq. yd.; $8 x 12 = $96



The area consists of 12 squares (12 sq. yds.)
AREA OF RECTANGLE = length x width or
base x height

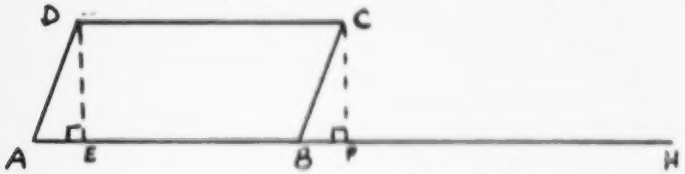
3. Ask: "What is the area of another room whose length and width are each <u>double</u> the dimensions of the bedroom?" (48 sq. yds. or FOUR TIMES the original area.)
What will the area be if the length is multiplied by 2 and the width by 3? (new area = 72 or 6 times original)

4.



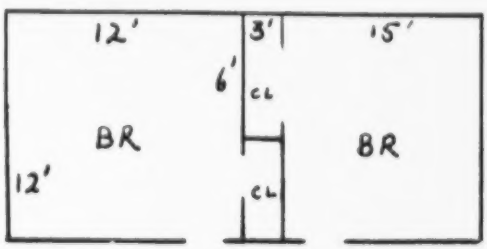
Ask: "How can we find the area of <u>parallelogram</u> ABCD?" (Students may respond AB·CB. If so, demonstrate that the area will be smaller than this when m∢A < 90° by collapsing ∢A to 0.) Can we form a rectangle of equal area to ABCD?

Extend $\overline{AB}$ through B (to H) and add segments $\overline{DE}$ and $\overline{CF}$ to the figure, each perpendicular to $\overline{ABH}$.



(Demonstrate with cutouts that △AED can be removed and placed over △BFC creating rectangle EFCD equal in area to ABCD. Why are these triangles congruent?) Therefore the area of parallelogram ABCD also equals base x height. (The <u>height</u> of the parallelogram is represented by segments $\overline{DE}$ or $\overline{CF}$, and is shorter than side $\overline{AD}$, unless ∢A is a right angle.)

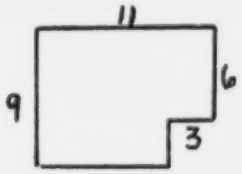
APPLICATIONS:



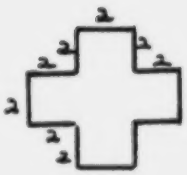
1. Find the area in square feet of each bedroom and closet in this floor plan.
   (12x12 = 144 sq. ft., 3x6 = 18 sq. ft., 12x15 = 180 sq. ft.)

2. If both bedrooms are to be covered with carpet costing $12.50 per sq. yd., find the total cost.
   (12' = 4 yd., 15' = 5 yd., total area = 16+20 = 36 sq. yd., cost = $450 or
   total area = 324 sq. ft. = $\frac{324}{9}$ = 36 sq. yd.

3. Find the area of each region.

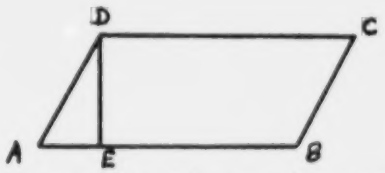


(72+18 = 90) or
(99-9 = 90)



(5x(2x2) = 20) or
(36-4(2x2) = 20)

4. Find the area of parallelogram ABCD if AB = 8", BC = 4", DE = 3". (24)



5. If the area of parallelogram ABCD equals 32 sq. in. and the height is 4", find the length of the base ($\frac{32}{4}$ = 8").

6. Parallelogram LMNO has an area of 12 sq. in. Find the new area if:
   a) the height is doubled (24).
   b) the base was cut in half (6).
   c) the height was doubled and the base was halved (12).
   d) the base and height were both tripled (108).
   e) the base doubled and the height tripled (72).

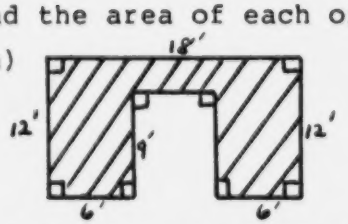
SUMMARY:

1. What is the area of a basketball court 50' by 84'? (4200 sq. ft.)
2. How can we find the area of a parallelogram? (base x height)
3. What is meant by the <u>height</u> of a parallelogram?

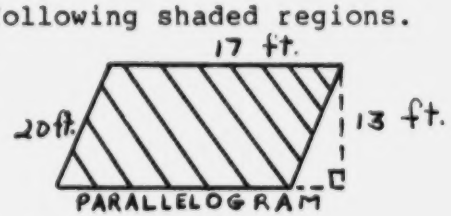
HOMEWORK:

1. Find the area of each of the following shaded regions.

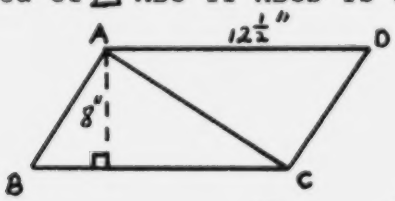
a)



b)



2. Find the area of △ ABC if ABCD is a parallelogram.



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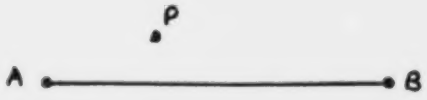
3. Show that a diagonal of a parallelogram splits the parallelogram into two congruent triangles.

4. Show how to bisect $\overline{AB}$ without using intersecting arcs below $\overline{AB}$. Justify.

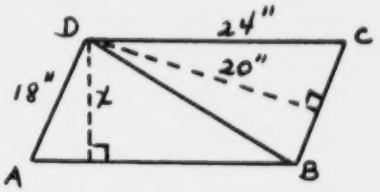
5. Show how to construct a line through P $\perp$ $\overline{AB}$ without using intersecting arcs below $\overline{AB}$.



6. If the perimeter of a rectangle is 48', what must its length and width be for the area to be the greatest? Try to justify your answer.

7. If the area of $\boxed{P}$ ABCD is 288 sq. ft. and AB = 15 ft., what is its height as measured to $\overline{AB}$?

8. Find the height of $\boxed{P}$ ABCD to $\overline{AB}$ using the measurements shown in the figure.



9. A rectangle has a perimeter of P in. and an area of A sq. in. It is enlarged by a factor of 5, so its length and width are each multiplied by 5. How are its perimeter and area changed?

10. The length of a rectangle is three less than twice its width. If the length is increased by 3 and the width is decreased by 1, a new rectangle is formed whose area is 5 more than the original rectangle. Find:
    a) the dimensions of the original rectangle.
    b) the area of the new rectangle.

Lesson 22 - Algebra and Geometry

AIM: How do we find the area of a trapezoid or a triangle?

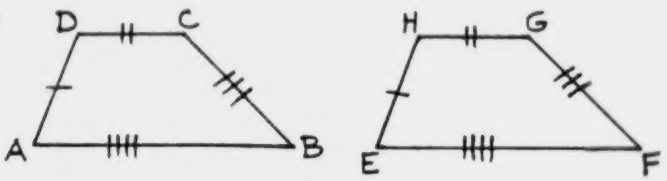
PERFORMANCE OBJECTIVES: The student will be able to...

1. Find the area of a trapezoid.
2. Find the area of a triangle.

VOCABULARY: no new terms

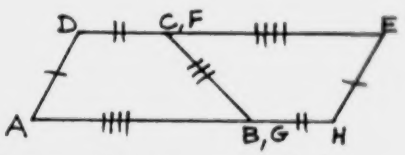
MOTIVATION:

Combine trapezoids ABCD and EFGH to form a parallelogram.

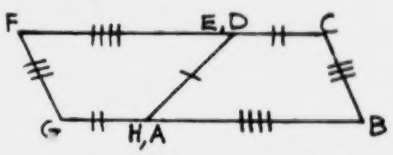


DEVELOPMENT:

1. Discuss motivation. Cut-out trapezoids will help students visualize the situation. There are two ways to solve the problem:
   a) rotate EFGH and move it so that $\overline{GF}$ coincides with $\overline{BC}$. $\overline{AD} \cong \overline{EH}$, $\overline{AH} \cong \overline{DE}$ (Elicit the reason: sums of equals are equal.) Therefore, AHED is a parallelogram.



   b) Place the trapezoids so that $\overline{AD}$ coincides with $\overline{HE}$.



2. Ask, "How is the area of (either) parallelogram related to the areas of the original trapezoids?" (their sum) Ask, "How are the areas of the two trapezoids related?" (Since all corresponding parts are =, and since the congruent polygons coincide when placed one upon the other, it is reasonable to say that the areas are equal.) Since Area of Parallelogram = 2 (Area of Trapezoid), we can say that Area of Trapezoid = $\frac{1}{2}$ Area of this Parallelogram.

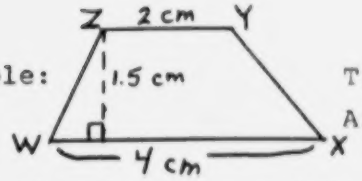
3. Since we know how to find the area of a parallelogram, and we now have a situation which relates the area of a trapezoid to the area of this parallelogram, we can try to derive a formula for the area of a trapezoid. Introduce notation to simplify and generalize the discussion: b_1 and b_2 to represent the bases of the trapezoid and h as its height.

If we let b_1 = AB and b_2 = CD, with h = height, what is the area of AHED? (base = b_1+b_2, area = $h(b_1+b_2)$)

If the two trapezoids are congruent, what must be the area of each? ($\frac{1}{2}h(b_1+b_2)$)

AREA(TRAPEZOID) = $\frac{1}{2}h(b_1+b_2)$ = $h(\frac{b_1+b_2}{2})$ = the height times the average of the bases

2. Example:



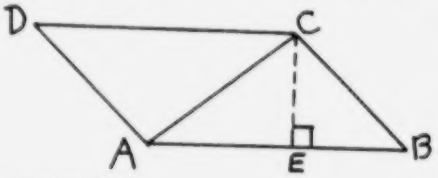
Trapezoid WXYZ

Area = $\frac{1}{2}(1.5)(2+4)$ = 4.5 sq. cm

3. (Have the class do a similar problem from the Applications section.)

4. Ask, "How can we use our knowledge of the parallelogram to find the area of a triangle?" Elicit: A diagonal divides a parallelogram into two congruent triangles.

Given parallelogram ABCD... since $\triangle ABC \cong \triangle CDA$, each has half the area of the parallelogram.

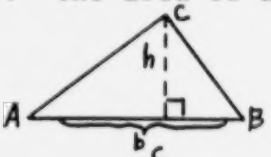


Area ABCD = AB x CE (base x height)

Area ABC = $\frac{1}{2}$(ABxCE)

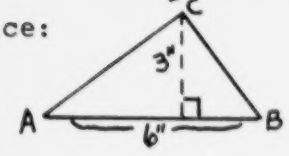
But $\overline{AB}$, $\overline{CE}$ are also the base and altitude of $\triangle ABC$.

Conclude: the area of a triangle is $\frac{1}{2}$ x base x height.



AREA(TRIANGLE) = $\frac{1}{2}bh$

5. Practice:

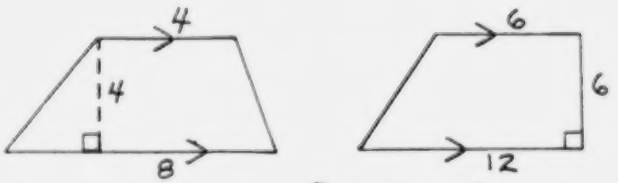


area of $\triangle ABC$ = $\frac{1}{2}(6)(3)$ = 9 sq. in.

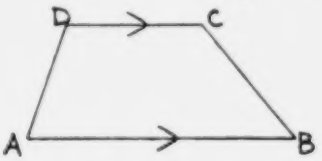
If the area of a triangle is 45 sq. cm and the height 6, how long would the base be?
(area = $\frac{1}{2}$bh, 45 = $\frac{1}{2}$b(6), 45 = 3b, b = 15)

APPLICATIONS:

1. Find the area of each trapezoid. (24, 54)



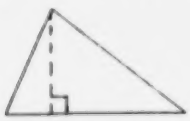
2. a) If AB = 14", CD = 8" and the area is 110 sq. in., find the height.
(110 = $\frac{1}{2}$(x)(14+8), 110 = 11x, x = 10)



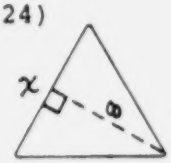
b) If AB = 12, height = 5 and area = 50, find CD.
(50 = $\frac{1}{2}$(5)(12+x); 100 = 60 + 5x, 5x = 40, x = 8)

c) If the upper base of a trapezoid is twice as long as the lower base, the height is 4 and the area is 72, find the length of each base.
(72 = $\frac{1}{2}$(4)(x+2x), 72 = 2(3x) = 6x, x = 12
lower base = 12, upper base = 24)

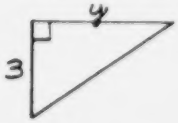
3.



h = 2, b = $4\frac{1}{2}$
find area ($4\frac{1}{2}$)



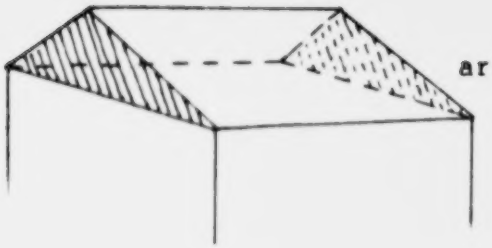
area = 20, find x
(20 = $\frac{1}{2}$(x)(8), x = 5)



area = 6
find y (4)

find area (12)

4. Two ends of a barn roof are in the shape of triangles, base = 40', height = 15'. Each end is to be painted with a paint for which 1 gallon covers 400 sq. ft. If each end is painted, how many gallons will be needed?



area of each triangle = $\frac{1}{2}bh$

$= \frac{1}{2}(40)(15) = 300$

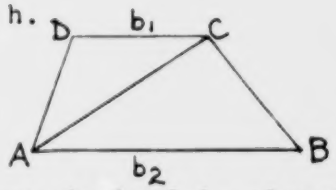
total area = 600 sq. ft.

gallons needed = $\frac{600}{400} = 1.5$

SUMMARY:

1. How do we find the area of a triangle? ...a trapezoid?

2. ABCD is a trapezoid whose height is h.



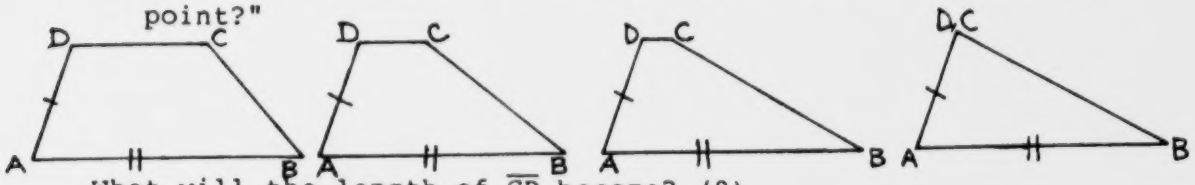
What is the area of △ABC? ($\frac{1}{2}b_2h$)

What is the area of △ADC? ($\frac{1}{2}b_1h$)

Area ABCD = $\frac{1}{2}b_1h + \frac{1}{2}b_2h$

How is this formula related to the one derived in class? (equivalent)

3. In trapezoid ABCD, what happens if $\overline{CD}$ shrinks to a single point?"



What will the length of $\overline{CD}$ become? (0)

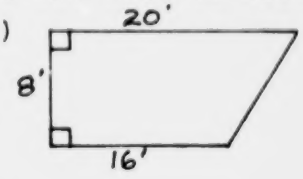
What kind of figure will we have? (△ABD)

In the area formula for a trapezoid, what is the result if a base equals zero? ($\frac{1}{2}h(b_2)$ or $\frac{1}{2}bh$, the area formula for a triangle.)

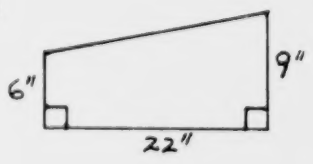
HOMEWORK:

1. Find the area of each trapezoid.

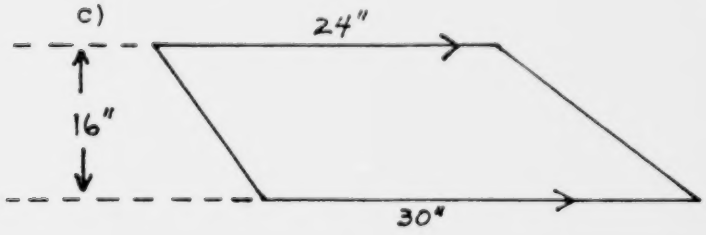
a)



b)

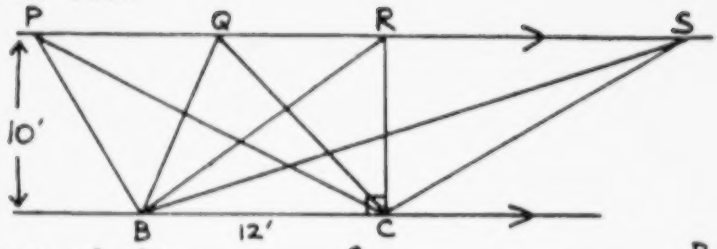


c)

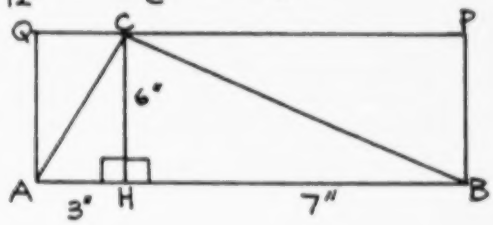


2. What is the area of each triangle?

   a) △PBC
   b) △QBC
   c) △RBC
   d) △SBC



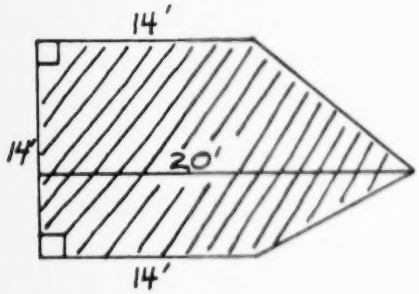
3. By finding the area of the smaller right triangles, △AHC and △CHB, show that the area of △ABC is one-half the area of the rectangle ABPQ.



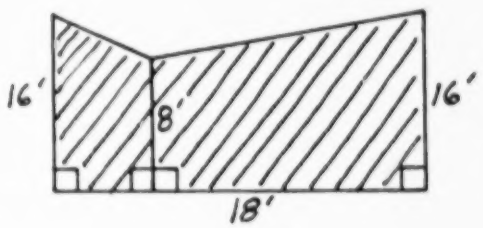
4. Show (without using numbers) that when the altitude falls inside the triangle, as in ex. 3 above, that the area of the triangle will always be one-half the area of the rectangle.

5. Find the area of each shaded region.

   a)



   b)



Lesson 23 - Algebra and Geometry

AIM: What is meant by the volume of a solid?

PERFORMANCE OBJECTIVES: The student will be able to...

1. Define volume.
2. Identify rectangular solids.
3. Find the volume of a rectangular solid.

VOCABULARY: volume, rectangular solid, face, edge, vertex, surface

MOTIVATION:

What factors must be considered in determining the best size of a room air conditioner?

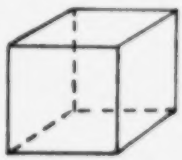
(kind of room, e.g., kitchen or bedroom?
location, e.g., top floor or bottom floor?
size)

How should we measure the size of a room?

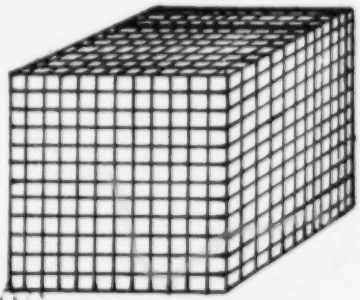
DEVELOPMENT:

1. Informally, volume is the amount of space occupied by a solid figure. More formally, it is the number of cubic units (cu. in., cu. cm, etc.) taken up by the solid figure.

   Examine a cube. How many <u>vertices</u> does it have? (eight)
   Each <u>face</u> is a square.
   How many faces does it have? (six)
   All the <u>edges</u> are equal in length.
   How many edges does it have? (twelve)

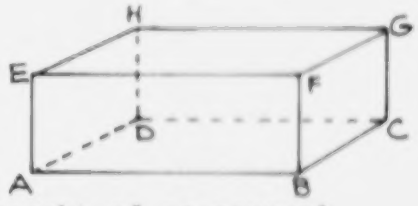


   One cubic inch is the space contained within a cube 1" wide, 1" long and 1" high. How many cubic inches are in a cubic foot? A cubic foot has 12" in length, 12" in width and 12" in height. It can be thought of as



consisting of 12 layers of 1" blocks, each layer 12 blocks long and 12 blocks wide and 1" thick. Therefore, each layer has 12 x 12 or 144 cubes 1" thick; the entire solid has 12 such layers or 1728 cubes. One cubic foot therefore contains 1,728 cubic inches.

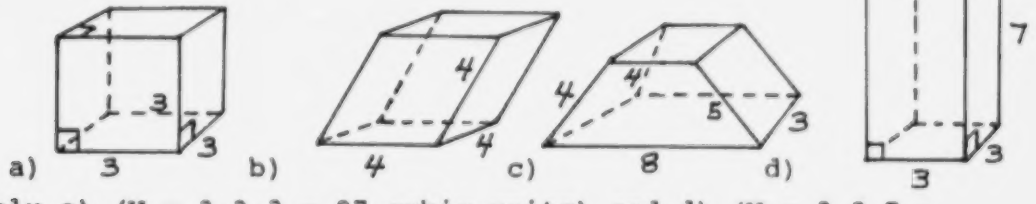
2. A solid figure is a <u>rectangular solid</u> if each face is a rectangle. The six faces here are: ABCD, ABFE, BCGF, CDHG, DAEH AND EFGH



Since a rectangular solid can be thought of as several layers of blocks, each layer with the shape of a rectangle, the total number of blocks (or cubic units) is the number of layers (height) times the number of blocks in each layer. Because each layer is a rectangle, each contains length x width blocks. The total number of blocks or cubic units is length x width x height. Or, Volume = l·w·h.

3. Find the volume of a box 4" long, 5" wide and 3" high (60 cu. in.).

4. Identify which of the following are rectangular solids and find their volumes.



Only a) (V = 3·3·3 = 27 cubic units) and d) (V = 3 3·7 = 63 cubic units).

APPLICATIONS:

1. The strength of an air conditioner is measured in BTU (British Thermal Units).
   Required BTU = 4.5 x volume (in cu. ft.) + adjustment.
   Adjustment: top floor: +1000
   kitchen: +1500
   living room: +1000

If a top floor bedroom measures 10' by 14' with an 8' ceiling, how many BTU are needed?

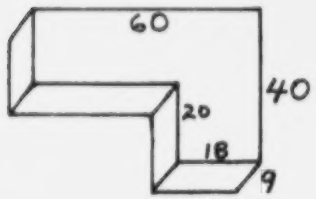
Volume = 10 x 14 x 8 = 1120 cu. ft.

BTU = 4.5(1120) + 1000 = 5040 + 1000 = 6040 BTU

How many BTU for a first floor living room 15' x 18' with an 8' ceiling?

(4.5 x (15x18x8) + 1000 = 9720 + 1000 =10720 BTU)

2. A 9' deep foundation is to be dug for an "L-shape" house, with indicated dimensions. How much earth must be removed? Elicit that 2 rectangular solids may be used.



Volume = 60(40-20)x9 + (18x20x9)

= 10800 + 3240 = 14040 cu. ft.

3. A cereal box is 2" wide , 6" long and 8" high. A company vice president suggests putting twice as much cereal in each box by doubling each dimension. Will this work?

Original volume = 2x6x8 = 96 cu. in.

New dimensions: 4,12,16

New volume: 4x12x16 = 768 cu. in.

$\frac{768}{96}$ = 8 times original

(We double the volume by doubling one dimension, not all three.)

4. What is the difference in volume between a box 3x6x5 inches and a box 6x4x4 inches?

3x6x5 = 90

6x4x4 = 96, 96 - 90 = 6 cu. in.

Which will <u>look</u> bigger on the shelf?

SUMMARY:

1. Volume measures the space contained or occupied by a solid figure. In which application is volume a consideration?
   a) How much chlorine to put in a pool? (yes)
   b) How much paint for a room? (no, "wall space" or area)

c) How much fence for a yard? (no, perimeter or length)

d) What size air conditioner for a room? (yes)

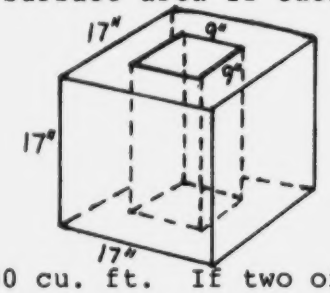
e) How much water for a potted plant? (yes)

2. If each dimension of a rectangular solid is <u>doubled</u>, how many times larger will the new volume be?

HOMEWORK:

1. A solid box measures 3' by 4' by 5'.
   a) What is its volume?
   b) What is the total area for all six faces?
   c) Find the new volume and total surface area if each dimension is <u>doubled</u>.

2. What is the volume of the solid with a hole as shown in the figure, a box with a box-cutout?



3. The volume of a box is given as 8,000 cu. ft. If two of its dimensions are 18' and 40', what is its third dimension?

4. Water weighs $62\frac{1}{2}$ lbs. per cubic foot. A container in the shape of a box measures 30" by 30" by 18" and is half full of water. How much does the water weigh?

5. A column of mercury measures $\frac{1}{8}$" by $\frac{1}{8}$" by 12". If mercury weighs about 13.5 times as much as water, how much will such a column of mercury weigh? Use the fact that 1 cu. ft. of water weighs about $62\frac{1}{2}$lbs.

6. How many cubic inches are there in a cubic yard?

7. A container in the shape of a box but open at one end is to contain 108 cu. in. If the base is a square, what dimensions will use the least amount of material to make the container. Hint: Try bases measuring 4" x 4", 6" x 6", 8" by 8".

8. A conversion table reads "to convert from cubic inches to liters multiply by .016387". Using this information only, about how many cubic inches are there in 1 liter? (Find the answer to the nearest cubic inch.)

9. There are 231 cubic inches in one gallon. How many gallons are there in 1 cu. ft.? (Find the answer to the nearest gallon.)

10. One liter contains about 61 cubic inches. About how many liters are there in one cubic foot?

11. Using the information given in ex. 9 and 10, about how many liters are there in one gallon? (Find the answer to the nearest liter.)

12. Which is a better buy, $1.25 a gallon or 35¢ a liter for the same gasoline?

Lesson 24 - Algebra and Geometry

AIM: How do we find the volume of a prism?

PERFORMANCE OBJECTIVES: The student will be able to...

1. Identify prisms.
2. Find the volume of a prism.

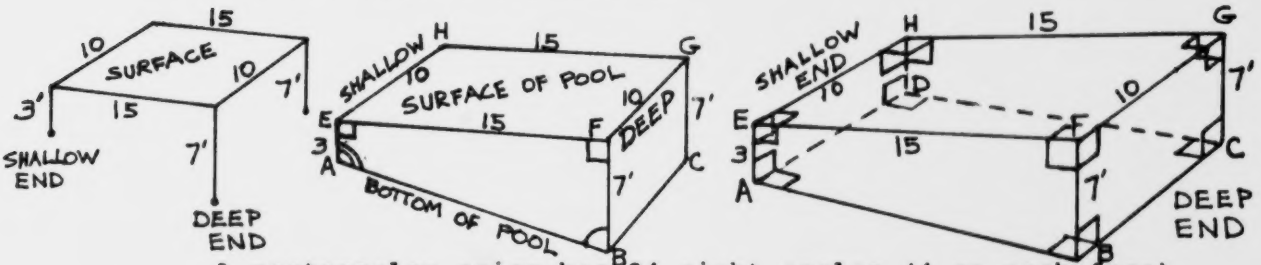
VOCABULARY: prism

MOTIVATION:

A swimming pool begins with a depth of 3' and gradually increases to a 7' depth. If the surface is 10' wide and 15' long, how much water is needed to fill the pool?

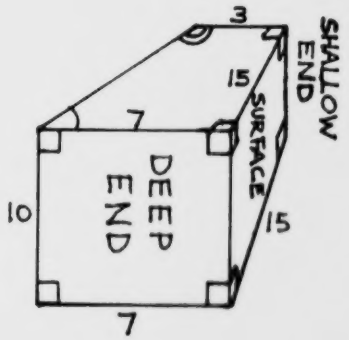
DEVELOPMENT:

1. Discuss motivation. Elicit that the solid is not a rectangular solid. Elicit that a diagram would be useful. Elicit the diagram:



A rectangular prism has 24 right angles (4 on each face). This solid has only 20 right angles on its faces. How many faces are rectangles? (four) Which faces are rectangles? (EFGH, FGCB, EHDA, ABCD) Which faces are not rectangles? (ABFE, DCGH) What are they? (trapezoids)

Students may visualize the solid more clearly if they imagine this block of water placed on its side. (View from the deep end.)



This solid is a member of a set of solids which also includes rectangular solids. The larger set is the set of prisms.

Define prism: a solid figure bounded by polygons. Its two bases must be congruent polygons in parallel planes; the non-bases (lateral faces) must be parallelograms.

In our example, trapezoids ABFE and DCGH are the bases. The lateral faces are parallelograms ABCD, BCGF, EFGH and ADHE (which are, in fact, rectangles). When the lateral faces of a prism are rectangles, the prism may be called a right prism.

Ask, "What is the distinction between a rectangular solid and a rectangular prism?" (In a rectangular solid, all six faces must be rectangles. In a rectangular prism, one pair of opposite faces (the bases) may be rectangles or some other congruent polygons.)

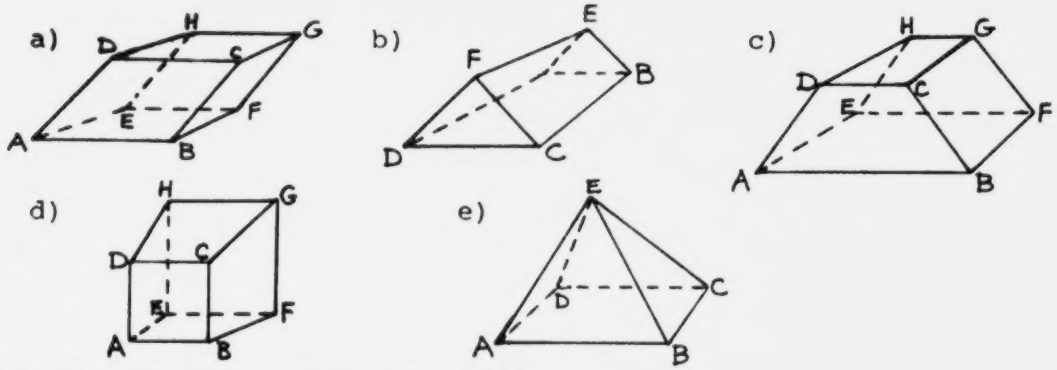
Our problem with the swimming pool is a particular instance of the general problem of finding the volume of a prism. We will attack the pool problem by considering the general case, and finding a method which works for all prisms.

2. Elicit that the volume of a rectangular solid (which is always a type of prism) equals lwh (from lesson #23). If we consider lw to be the area of a "base" of the prism and h the altitude between bases, then the volume for a rectangular solid is Base area (B) x Altitude (h). This formula can be used for any prism.

3. In our pool problem, the area of a base (trapezoid) may be found by the formula $A = \frac{1}{2}h(b_1+b_2)$. The Area of Base ABFE = $\frac{1}{2}(15)(3 + 7) = 75$. The altitude between the bases measures ten feet. Therefore, the volume equals $B \cdot h = 75 \cdot 10 = 750$ cubic feet.

APPLICATIONS:

1. Which of the following are prisms?



a), b), c) yes, congruent bases are:

a) Any pair of parallel faces (This is a parallelepiped.)

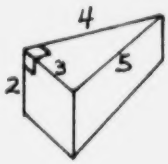
b) DCF, ABE (triangles)

c) ABCD, EFGH (trapezoids)

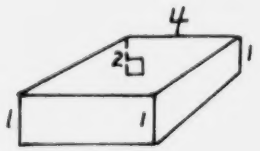
d) No, parallel bases (squares) are <u>not congruent</u>.

e) No (pyramid)

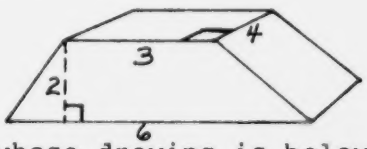
2. Find the volume of each prism.



a) base area = $\frac{1}{2}(3 \cdot 4) = 6$
altitude = 2
volume = 12 cubic units

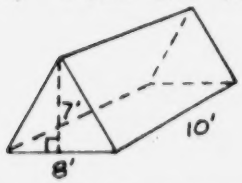


b) base area = $4 \cdot 1 = 4$
altitude = 2, V = 8
or: base area = $4 \cdot 2 = 8$
altitude = 1, V = 8 cubic units



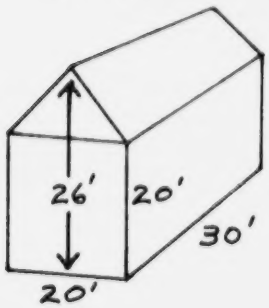
c) base area = $\frac{1}{2}(2)(3 + 6) = 9$
altitude = 4
volume = 36 cubic units

3. a) Find the volume of the tent whose drawing is below.



base = $\frac{1}{2} \cdot 7 \cdot 8 = 28$
altitude = 10
V = 280 cu. ft.

b) Find the volume of the barn.



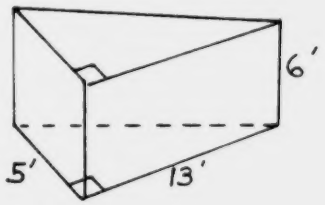
base = front face
base area = square + triangle
= $(20 \times 20) + (\frac{1}{2} \times 6 \times 20) = 460$
altitude between bases = 30
Volume = 13,800 cu. ft.

SUMMARY:

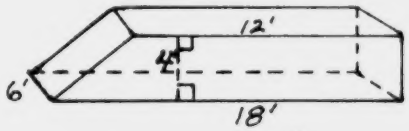
1. Write a description of what a prism is.
2. Draw a prism with triangular bases.
3. How could we tell which solids were <u>not</u> prisms? (If parallel bases can't be found, or if the bases are not congruent, the solid is not a prism.)

HOMEWORK:

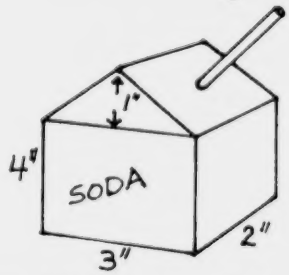
1. A coal bin has the shape of a triangular prism as shown. What is its volume?



2. A tank has the shape of a prism shown at the right. What is its volume?



3. A soda pop container in the shape of a prism is shown at the right. How many cubic inches of soda can it hold?



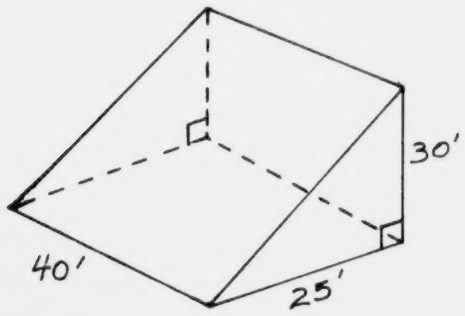
4. The following box containers have the <u>same</u> <u>total</u> <u>surface area</u>. Which one has the greatest volume? What is this common surface area?
   a) 2' x 6' x 12' b) 3' x 6' x 10' c) 6' x 6' x 6'
   d) Try to make a generalization based on your answers to a, b, c.

5. The following box containers have the <u>same</u> <u>volume</u>. What is this volume? Which has the least total surface area?
   a) 3" x 24" x 24" c) 9" x 12" x 16"
   b) 4" x 18" x 24" d) 12" x 12" x 12"
   e) Try to make a generalization based on the above answers.

6. Find the total surface area of the containers for

   a) ex. 1 b) ex. 2 c) ex. 3

7. A building with a solar roof has the shape of a prism. What is its volume?



Lesson 25 - Rational and Irrational Numbers

AIM: What are rational and irrational numbers?

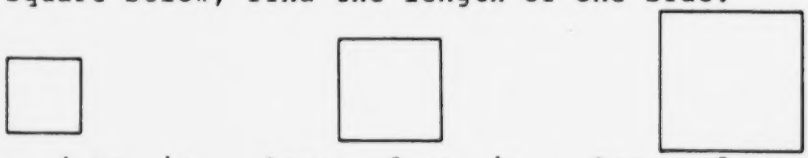
PERFORMANCE OBJECTIVES: The student will be able to...

1. Define a rational number.
2. List the types of numbers that can be written as a fraction (whole numbers, fractions, terminating decimals, nonterminating repeating decimals).
3. Define irrational number.
4. Give examples of rational and irrational numbers.

VOCABULARY: rational number, irrational number, terminating decimal, nonterminating repeating decimal, nonterminating nonrepeating decimal, real number

MOTIVATION:

For each square below, find the length of one side.



Area = 4 sq. in. Area = 6 sq. in. Area = 9 sq. in.

DEVELOPMENT:

1. Discuss motivation. We are looking for the positive root of $x^2 = 4$, $x^2 = 6$ and $x^2 = 9$. The first square has a 2" side; the third square has a 3" side. The center square must have a side whose length is between 2" and 3" since the squares could be nested like this:

Elicit that we may try to "trap" the side of the square between two numbers which are less than one inch apart. Ask, "Will the side of the middle square be longer or shorter than 2"?" Since $(2\frac{1}{2})^2 = (2\frac{1}{2})(2\frac{1}{2})$ or $(2.5)(2.5) = 6.25$ or $6\frac{1}{4}$, it is clear that the side will be shorter than $2\frac{1}{2}$". Try several other approximations suggested by the students. Ask, "If we

continue with this method, do you think we will find the exact answer? Does an exact answer exist?" (Yes: The square is real.)

The length of the side of the middle square may be designated $\sqrt{6}$ (read: the square root of 6). This number is approximately equal to 2.4494897.

2. Define rational numbers.

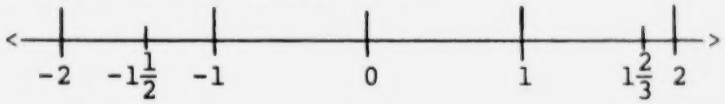
Rational numbers are numbers that can be expressed in the form $\frac{a}{b}$ where a and b are integers, $b \neq 0$. Numbers that are rational are whole numbers, natural numbers, integers, fractions and mixed numerals. For example, 5, -1, $\frac{7}{8}$, $6\frac{2}{3}$ and 4.2 are rational numbers.

3. State the properties of the set of rational numbers.

a) The set of rational numbers is closed under addition, subtraction, multiplication and under division by nonzero rational numbers. This means that whenever you take the sum, difference, product or quotient (with a divisor other than zero) of two rational numbers, the result will be a rational number. For example: -8 divided by 3 is $\frac{-8}{3}$, which is a rational number. ($-2\frac{2}{3}$ or -2.6666...)

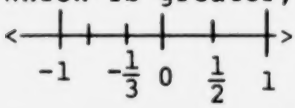
b) Every nonzero rational number has a multiplicative inverse. For example, $\frac{2}{5} \times \frac{5}{2} = 1$.

c) Every member of the set of rational numbers may be located on a number line.



d) Show ordering of rational numbers on the number line. Given any two unequal rational numbers, we can determine which is greater.

Which is greater, $\frac{1}{2}$ or $-\frac{1}{3}$?



We see that $\frac{1}{2}$ is to the right of $-\frac{1}{3}$.

Answer: $\frac{1}{2} > -\frac{1}{3}$.

e) The set of rational numbers is "dense". This means that given any two unequal rational numbers we can

find a rational number between them. For example, $\frac{1}{8}$ and $\frac{1}{2}$. $(\frac{1}{8} + \frac{1}{2}) = \frac{5}{8}$, $\frac{5}{8} \div 2 = \frac{5}{16}$. $\frac{5}{16}$ is a rational number which is between $\frac{1}{8}$ and $\frac{1}{2}$.

3. A rational number may be written as a decimal. To change a rational number to a decimal, we perform the indicated division:

$\frac{1}{2}$ $\frac{.500}{)1.000}$, $\frac{3}{4}$ $\frac{.75}{)3.00}$, $\frac{1}{16}$ $\frac{.0625}{)1.0000}$

Terminating decimals "end". We eventually obtain only zeros as new digits in the quotient.

Other rational numbers lead to a repeating pattern of digits:

$\frac{1}{3}$ $\frac{.333}{)1.000} = .\overline{3}$ $\frac{2}{11}$ $\frac{.1818...}{)2.0000} = .\overline{18}$ $\frac{1}{6}$ $\frac{.166}{)1.000} = .1\overline{6}$

Put the bar over the repeating part. The same digits repeat endlessly. These are called nonterminating repeating decimals. Every rational number is either a terminating decimal or a repeating decimal. Rationals can be positive or negative. If the bar is there, it is rational.

4. Do Applications 1, 2, 3.

5. Students should be able to express a decimal as a fraction, a symbol for a rational number (place a decimal in "rational form").

$.4 = \frac{4}{10}$ $.26 = \frac{26}{100}$ $.341 = \frac{341}{1000}$ $.0624 = \frac{624}{10000}$

6. Do Application 4.

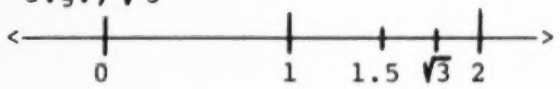
7. Define irrational numbers.

A decimal that doesn't repeat such as .080080008... is not repeating and not terminating. It is not rational. It is irrational - an irrational number may be represented by a nonrepeating nonterminating decimal. You are not always sure what the next digit will be.

$\sqrt{6}$ is irrational. It is between 2 and 3. π is irrational. $\pi = 3.14159...$ It cannot be written as a terminating decimal or as a nonterminating repeating decimal.

8. Do Application 5.

9. Irrational numbers are real numbers. Students should be able to visualize irrational numbers as points on the number line. e.g., $\sqrt{3}$



10. Irrational numbers may be approximated by rational numbers. For example, $x = .38541... \approx .39$.

11. Define real numbers: The set containing all rational numbers and all irrational numbers is the set of <u>real numbers</u>.

APPLICATIONS:

1. State which of the given numbers is greater.
   a) $\frac{5}{3}, \frac{7}{3}$
   b) $-\frac{2}{3}, \frac{5}{3}$
   c) $-\frac{3}{4}, -\frac{4}{3}$
   d) $1.5, 1\frac{3}{5}$
   e) $-3.6, -3\frac{3}{4}$

2. Find a rational number between $\frac{1}{4}$ and $\frac{1}{2}$.

3. Write each rational number as a repeating decimal:
   $\frac{2}{5}, \frac{5}{6}, \frac{3}{10}, \frac{4}{7}, \frac{1}{8}$

4. Express each of the following rational numbers as a fraction: $.5, .0\overline{4}, -.4$

5. Tell whether the number is rational or irrational or whether there is not sufficient information to tell.

.52	$-.\overline{621}$
.525252...	π
$.5\overline{2}$	$.7\overline{4}$
.525525552...	3
.24389216...	.13121110987...
.124124124...	6.25
$.525525\overline{52}$	$6 + \pi$

SUMMARY:

1. How is the set of whole numbers related to the set of rational numbers?

2. How do we define a rational number in terms of integers?

3. Why is an integer considered to be a rational number?

4. Give an example of an irrational number.

5. State whether the statement is true or false:
   a) Every rational number corresponds to a point on the number line.
   b) There are some numbers that are both rational and irrational.
   c) Every irrational number corresponds to a point on the real number line.
   d) Every real number is rational.
   e) Every irrational number is real.

HOMEWORK:

1. State which of the given numbers is greater.

   $\frac{1}{3}$ or -2 $\quad$ $\frac{5}{8}$ or $\frac{-12}{8}$ $\quad$ $\frac{-10}{4}$ or $\frac{-13}{4}$

2. Name a rational number which lies between the two given numbers.

   $\frac{1}{3}$ and $\frac{7}{9}$ $\quad$ -2 and -3 $\quad$ $-2\frac{1}{3}$ and -3

3. Write each rational number as a repeating decimal:

   $\frac{11}{5}$ $\quad$ $\frac{7}{9}$ $\quad$ $\frac{2}{9}$ $\quad$ $\frac{1}{11}$

4. Tell whether the number is rational or irrational or whether there is insufficient information to tell.

5	.45	1.265
9	.454454445...	1.265438...
$.4\overline{5}$	.4041	1.26555...

5. Express as a quotient of two integers:

a)	4.3	e)	$.1\overline{3}$	i)	1.4
b)	$.08\frac{1}{4}$	f)	$.3\overline{9}$	j)	$1.\overline{4}$
c)	$.2\frac{1}{3}$	g)	$.\overline{4}$	k)	$1.0\overline{4}$
d)	$-.07$	h)	$.0\overline{4}$	l)	$1.\overline{04}$

6. State whether each of the following is rational, irrational or insufficient information to tell:

a) 2.3

b) $2.\overline{3}$

c) 2π

d) $2 + \pi$

e) .1010010001... (The number of zeros increases by 1 in each group.)

f) .12321...

g) -17

7. Answer TRUE or FALSE or NOT LEARNED YET.

a) Every real number has a location point on the number line.

b) Every point on the number line names some real number.

c) $\sqrt{2}$ is an irrational number.

d) The sum of two rational numbers is always a rational number.

e) The sum of two irrational numbers is always an irrational number.

f) The product of two rational numbers is always a rational number.

g) The product of two irrational numbers is always an irrational number.

h) The sum of a rational number and an irrational number is always an irrational number.

Lesson 26 - The Square Root of a Number

AIM: How do we find the square root of a number?

PERFORMANCE OBJECTIVES: The student will be able to...

1. Estimate the square root of positive integers less than 100 to the nearest tenth.
2. Compare positive and negative square roots of perfect squares.

VOCABULARY: index, radical, radical sign, radicand, principal square root, perfect squares

MOTIVATION:

1. If the area of a square is 25, find the length of one side.
2. What is the square root of 25?
3. Find $\sqrt{25}$.
4. Solve for x: $x^2 = 25$

What do these problems have in common? How are they different?

DEVELOPMENT:

1. Discuss motivation. The length of a side must be 5 units. To find the square root of a number is to find one of its two equal factors. $\sqrt{25} = 5$ because $5 \cdot 5 = 25$ or $5^2 = 25$. $\sqrt{\ }$ is a <u>radical sign</u>. $\sqrt{25}$ is a <u>radical</u>. 25, the number under the radical sign, is called the <u>radicand</u>.
   $\sqrt{25}$ could be written as $\sqrt[2]{25}$. The number 2 is called the index. The index indicates how many equal factors we wish to find.
   The square of a positive <u>or</u> negative rational number is a positive number. So, $5^2 = 25$ or $(-5)^2 = 25$ because $5 \times 5 = 25$ and $(-5)\times(-5) = 25$, and the equation $x^2 = 25$ has two solutions: 5 and -5.

Numbers such as 25 are called perfect squares because they are squares of integers. Since every perfect square except zero is the product of two positive or two negative equal numbers, every positive perfect square has two square roots - a positive square root and a negative square root. However, we call the positive square root of a number its principal square root. $\sqrt{25} = 5$, the principal square root of 25. We write $\sqrt{25} = 5$ and use $-\sqrt{25}$ to denote -5 or $\pm\sqrt{25}$ when we wish to indicate both numbers, +5 and -5, which may also be written ± 5.

2. Students should know the perfect squares up to 144. Ask them to list the squares of numbers 1 through 12.

3. Do applications 1 and 2.

4. Suppose n is positive and not a perfect square or a quotient of perfect squares. Then what kind of number is $\sqrt{n}$? It is irrational and can be represented by a non-terminating nonrepeating decimal. We will approximate such numbers to the nearest tenth.
For example, consider $\sqrt{15}$. Let us approximate. Since $3^2 = 9$ and $4^2 = 16$, then $\sqrt{15}$ lies between 3 and 4. It is closer to 4. We try 3.9; $3.9^2 = 14.21$; $3.9 < \sqrt{15} < 4.0$; since 15 is closer to 14.21 than to 16.00, we say $\sqrt{15} \approx 3.9$
Here is a second method: $\sqrt{15}$ is closer to 4. We divide 15 by 4. $4\overline{)15.00}$ $\;3.75$ Do not round off the quotient; carry it two more digits than the divisor. We know $\sqrt{15}$ is between 3.75 and 4. Take the average, $\frac{3.75+4}{2} = 3.87$. Divide 15 by 3.87. $3.87\overline{)15.00.00}$ $\;3.87$ $\sqrt{15} \approx 3.9$ to the nearest tenth.
Try another example, $\sqrt{52}$. It is between 7 and 8. It is closer to 7. Divide 52 by 7. $7\overline{)52.00}$ $\;7.42$ 52 is between 7 and 7.42. Take the average, $\frac{7+7.42}{2} = 7.21$. Divide 52 by 7.21. $7.21\overline{)52.00.00}$ $\;7.22$ So, $\sqrt{52} \approx 7.2$ to the nearest tenth.

5. Do Applications 3 - 5.

6. Ask, "Is it possible to find the square root of -36?"

Show there is no square root of a negative number in the set of real numbers. $\sqrt{-36}$ is not 6 because 6 x 6 = 36. It's not -6 because (-6)x(-6) = 36.
Advise the students that approximations of the square root of a rational number may be checked by using a table of squares and square roots.

APPLICATIONS:

1. Find the principal square root of the number: 1, 25, 81, 169, $\frac{1}{9}$, $\frac{9}{16}$, $\frac{49}{25}$, .81.
2. Express the radical as an integer, fraction or a decimal.
   $\sqrt{4}$, $\sqrt{36}$, $-\sqrt{100}$, $\sqrt{0}$, $\sqrt{400}$, $\sqrt{.49}$
   $-\sqrt{625}$, $\sqrt{\frac{1}{4}}$, $\pm\sqrt{\frac{9}{16}}$, $\sqrt{\frac{25}{81}}$, $\pm\sqrt{144}$
3. Approximate the square roots to the nearest tenth.
   80 , 14 , 3 , 21
4. Find the length of the side of the square whose area is 9.
5. Find the side of the square whose area is 8 to the nearest tenth.

SUMMARY:

1. How would you proceed to find an approximate value for $\sqrt{20}$?
2. Find the value of $(\sqrt{17})^2$. Explain.
3. Which is larger, $\sqrt{5}$ or $-\sqrt{5}$? Explain. Locate both numbers on the number line.

HOMEWORK:

1. Find the principal square root of: 9, 49, 121, 225.
2. Express the radical as an integer, fraction or decimal.
   $\sqrt{16}$ $\sqrt{64}$ $-\sqrt{144}$ $\pm\sqrt{196}$ $\sqrt{1}$
   $\sqrt{\frac{9}{4}}$ $\sqrt{.81}$ $\pm\sqrt{\frac{49}{100}}$ $\sqrt{36}$ $-\sqrt{81}$

3. Approximate the square root to the nearest tenth:

$\sqrt{2}$ $\sqrt{24}$ $\sqrt{90}$ $\sqrt{61}$

4. Find to the nearest tenth the length of a side of a square whose area is 17.

5. Find the principal square root of each of the following... to the nearest tenth if not a perfect square.

a)	256	d)	.49	g)	$.\overline{4}$
b)	2.56	e)	.4	h)	.16
c)	.81	f)	.09	i)	.1

6. Use a divide and average method to obtain the square root of each of the following to the nearest hundredth. Be sure you have a sound argument for the correctness of your answer, excluding a calculator or table.

a) $\sqrt{2}$ b) $\sqrt{3}$ c) $\sqrt{5}$ d) $\sqrt{.9}$

7. State in words the meaning of $\sqrt{n}$.

8. From the meaning of $\sqrt{n}$, what is the value of

a) $\sqrt{3} \times \sqrt{3}$ d) $\sqrt{6.25} \times \sqrt{6.25}$

b) $\sqrt{.7} \times \sqrt{.7}$ e) $\sqrt{.1} \times \sqrt{.1}$

c) $\sqrt{625} \times \sqrt{625}$ f) $\sqrt{a} \times \sqrt{a}$

9. Find the perimeter of a square if its area is

a) 900 sq. in.

b) 169 sq. yd.

c) 841 sq. mi.

10. Which is a better approximation to $\sqrt{6.4}$, 2 or 3? To verify your prediction, find $\sqrt{6.4}$ to the nearest tenth.

Lesson 27 - Multiplying Radicals

AIM: How do we multiply radicals?

PERFORMANCE OBJECTIVES: The student will be able to...

1. Multiply monomials containing radicals.
2. Simplify radicals.

VOCABULARY: review: radical sign, simplify, radicand, coefficient

MOTIVATION:

1. Find the area of a rectangle whose length is 8 and width is 2.
2. Find the area of a rectangle whose length is 8 and width is 2.

DEVELOPMENT:

1. Discuss the Motivation. Area = length x width = 8 x 2 = 16.

   Suppose the length and width were represented by $\sqrt{8}$ and $\sqrt{2}$. What would be the area of the rectangle?

   Since Area = length x width

   $$A = \sqrt{8} \times \sqrt{2}$$

   We can approximate the answer as follows:

   $\sqrt{2}$ is between 1 and 2

   $\sqrt{8}$ is between 2 and 3

   Therefore, the area will lie between (1)(2) and (2)(3); the area of the second rectangle is a number between 2 and 6. Using 1.5 as an approximation for the width and 2.5 as an approximation for the length, we obtain (1.5)(2.5) = 3.75 as an approximate value for the area. We can obtain a more accurate value by using better approximations of the width and length.

   Using the table of square roots $\sqrt{8} = 2.828$

   $\sqrt{2} = 1.414$

Multiply. $(2.828)(1.414) = 3.998792$ which when rounded off to the nearest hundredth becomes 4.00, equivalent to $\sqrt{16}$. This suggests that: $\sqrt{8}\sqrt{2} = \sqrt{16} = 4$ or in general: $\sqrt{a}\sqrt{b} = \sqrt{ab}$ where $a \geq 0$, $b \geq 0$.
Consider the special case when $a = b$: $\sqrt{5}\cdot\sqrt{5} = \sqrt{25} = 5$.
Generalize: $\sqrt{x}\cdot\sqrt{x} = x$ (In fact, this is the <u>definition</u> of x.)

2. If true, this rule predicts:
   $\sqrt{7}\cdot\sqrt{2} = \sqrt{14}$
   $\sqrt{7}\cdot\sqrt{4} = \sqrt{28}$
   $\sqrt{7}\cdot\sqrt{7} = \sqrt{49} = 7$ (true: definition of square root)
   $\sqrt{4}\cdot\sqrt{25} = \sqrt{100} = 10$ ($2\cdot5 = 10$: true)

3. Let us prove: $\sqrt{ab} = \sqrt{a}\sqrt{b}$ for $a \geq 0$, $b \geq 0$.
   Proof:
   We know $(\sqrt{ab})^2 = \sqrt{ab}\cdot\sqrt{ab} = ab$ (definition)
   Will $(\sqrt{a}\cdot\sqrt{b})^2 = ab$?
   Using the rearrangement property of multiplication:
   $(\sqrt{a}\cdot\sqrt{b})(\sqrt{a}\cdot\sqrt{b}) = \sqrt{a}\cdot\sqrt{b}\cdot\sqrt{a}\cdot\sqrt{b} = \sqrt{a}\cdot\sqrt{a}\cdot\sqrt{b}\cdot\sqrt{b}$
   But: $\sqrt{a}\cdot\sqrt{a} = a$, $\sqrt{b}\cdot\sqrt{b} = b$
   Hence: $(\sqrt{a}\cdot\sqrt{b})(\sqrt{a}\cdot\sqrt{b}) = ab$ or $(\sqrt{a}\cdot\sqrt{b})^2 = ab$
   Therefore: $\sqrt{a}\sqrt{b} = \sqrt{ab}$. So $\sqrt{ab} = \sqrt{a}\sqrt{b}$.

4. Apply the concept to the following examples:
   a) Find the area of a rectangle whose length is $\sqrt{6x}$ and width is $\sqrt{3x}$. Area = length x width
   $\sqrt{6x}\cdot\sqrt{3x} = \sqrt{18x^2}$
   b) Consider $\sqrt{7}\cdot\sqrt{5}\cdot\sqrt{2} = \sqrt{70}$.

5. Do Application 1.

6. Find the area of a rectangle whose length and width are $4\sqrt{5}$ and $3\sqrt{2}$.
   Use the commutative and associative properties of multiplication to develop rule:
   $(a\sqrt{b})(c\sqrt{d}) = a\cdot c\cdot\sqrt{b}\cdot\sqrt{d}$
   $= ac\cdot\sqrt{bd}$
   $(a\sqrt{b})(c\sqrt{d}) = ac\sqrt{bd}$ where $b \geq 0$ and $d \geq 0$
   Thus: $4\sqrt{5}\cdot3\sqrt{2} = 12\sqrt{10}$

If time permits, answer may be found to the nearest tenth. ($12 \cdot 3.16 = 37.92 \approx 37.9$)

Elicit the rule for multiplying two monomial square roots:

a) Multiply the coefficients.

b) Multiply the radicands.

c) If possible, simplify the results.

Consider: $3\sqrt{2} \cdot \sqrt{5} = 3\sqrt{10}$

$9\sqrt{a} \cdot 2\sqrt{a^3} = 18\sqrt{a^4} = 18a^2$

7. Do Application 2.

APPLICATIONS:

1. Multiply: $\sqrt{7}\ \sqrt{3}$ $\qquad$ $\sqrt{7}\ \sqrt{7}$

$\sqrt{4}\ \sqrt{4}$ $\qquad$ $\sqrt{2x}\ \sqrt{2x}$

$(-\sqrt{t})^2$ $\qquad$ $(\sqrt{5x})^2$

Simplify the results:

$\sqrt{5}\ \sqrt{4}$ $\qquad$ $\sqrt{6}\ \sqrt{8}\ \sqrt{3}$

$-\sqrt{2}\ \sqrt{8}$ $\qquad$ $\sqrt{3}\ \sqrt{10} \cdot \sqrt{3}$

Find the area of a square whose side is $\sqrt{3x}$.

Find the area of a rectangle whose length is $\sqrt{3y}$ and width is $\sqrt{2y}$.

Find the product and simplify: $\sqrt{18a}\ \sqrt{2a^3}$

$\sqrt{x} \cdot \sqrt{y} \cdot \sqrt{xy}$

2. Multiply and simplify:

$4\sqrt{3} \cdot 3\sqrt{12}$ $\qquad$ $3\sqrt{y} \cdot 2\sqrt{y}$

$4\sqrt{x^7} \cdot \sqrt{x^2}$ $\qquad$ $(3\sqrt{3})(2\sqrt{3})$

$(4\sqrt{5})(5\sqrt{3})(\sqrt{5})$ $\qquad$ $(3\sqrt{2})(2\sqrt{15})$

$(2\sqrt{3})^2$ $\qquad$ $(3\sqrt{2})(2\sqrt{3})$

$(2\sqrt{3})(5\sqrt{3})(\sqrt{8})$ $\qquad$ $(3\sqrt{x})(2\sqrt{y})(\sqrt{xy})$

Find the area in simplest form:

SUMMARY:

1. Given the area of a square, how do we express the length of the side of a square?
2. How do we multiply two monomial radicals?

HOMEWORK:

1. Multiply, then simplify the results.

$(2)(4\sqrt{7})$

$(5\sqrt{3})(3\sqrt{10})$

$(-\sqrt{y})^2$

$(5\sqrt{x})(3\sqrt{x})$

$4\sqrt{6}\cdot 2\sqrt{8}$

$(5\sqrt{x^2y})(7\sqrt{y^2})(\sqrt{y})$

$3\sqrt{2}\cdot 5\sqrt{2}$

$(\sqrt{6x})^2$

$(\sqrt{8})^2$

$(5\sqrt{6})(7\sqrt{4})$

$(2\sqrt{6})(3\sqrt{2})(\sqrt{3})$

2. Find the area of a square whose side is:

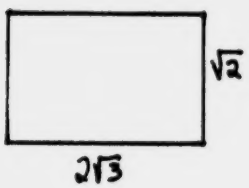
$\sqrt{2}$

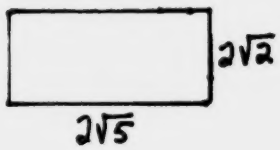
$3\sqrt{2}$

$\sqrt{17x}$

3. a) Find the area of the figure in simplest radical form.

   b) Approximate the length of the side correctly to the nearest tenth.





Lesson 28 - Simplifying a Square Root

AIM: How do we simplify square roots?

PERFORMANCE OBJECTIVES: The student will be able to...

1. Distinguish between finding a square root and simplifying a radical.
2. Simplify radicals.

VOCABULARY: simplify; review: radical, index, radicand, square root

MOTIVATION:

Evaluate each expression:

$\sqrt{4 \cdot 16}$ $\qquad$ $\sqrt{4} \cdot \sqrt{16}$

$\sqrt{9 \cdot 25}$ $\qquad$ $\sqrt{9} \cdot \sqrt{25}$

$\sqrt{25 \cdot 6}$ $\qquad$ $\sqrt{25} \cdot \sqrt{6}$

DEVELOPMENT:

1. Since $\sqrt{4 \cdot 16} = \sqrt{64} = 8$ and $\sqrt{4} \cdot \sqrt{16} = 2 \cdot 4 = 8$, then $\sqrt{4 \cdot 16} = \sqrt{4} \cdot \sqrt{16}$. Similarly, $\sqrt{225} = 3 \cdot 5 = 15$ and $\sqrt{150} = 5\sqrt{6}$. These examples illustrate the following property of square root radicals:

   The square root of a product of non-negative numbers is equal to the product of the square roots of the numbers.

2. If a and b are non-negative numbers, $\sqrt{a \cdot b} = \sqrt{a} \cdot \sqrt{b}$ where $a > 0$ and $b > 0$. This rule is used to <u>simplify a radical</u>, that is to transform it into an equivalent radical that does not contain a perfect square factor.

   For example: $\sqrt{12} = \sqrt{4 \cdot 3} = \sqrt{4} \cdot \sqrt{3} = 2\sqrt{3}$

   $\sqrt{48} = \sqrt{16 \cdot 3} = \sqrt{16} \cdot \sqrt{3} = 4\sqrt{3}$

   To simplify a radical:

   a) find 2 factors of the radicand, one of which is the largest perfect square factor of the radicand.

b) express the square root of the product as the product of the square roots of the factors.

c) find the square root of the factor that is a perfect square.

3. Do Application 1.

4. Distinguish between finding and simplifying a square root. Find $\sqrt{25}$. Simplify $\sqrt{48}$.

5. Do Application 2.

6. Find: $\sqrt{x^2}$, $\sqrt{x^4}$, $\sqrt{x^6}$

   Note: $a^m \cdot a^n = a^{m+n}$

   $a^2 \cdot a^2 = a^4$

   $a^3 \cdot a^3 = a^6$

   $a^4 \cdot a^4 = a^8$

   Simplify:

   $\sqrt{x^6y^2} = \sqrt{x^6}\sqrt{y^2} = x^3y$

   $\sqrt{25x^2} = \sqrt{25}\sqrt{x^2} = 5x$

   $\sqrt{64x^8y^{12}} = 8x^4y^6$

   $\sqrt{12a^4b^6c^8} = \sqrt{4}\sqrt{3}\sqrt{a^4}\sqrt{b^6}\sqrt{c^8} = 2\sqrt{3}a^2b^3c^4$

   $\sqrt{9x^2y^3} = \sqrt{9}\sqrt{x^2}\sqrt{y^3} = 3x\sqrt{y^2}\sqrt{y} = 3xy^2\sqrt{y}$

7. Do Application 3.

APPLICATIONS:

1. Simplify the expression:

$\sqrt{50}$	$\frac{1}{2}\sqrt{44}$	$3\sqrt{45}$
$\sqrt{27}$	$\sqrt{98}$	$2\sqrt{90}$
$3\sqrt{28}$	$-\sqrt{300}$	$\sqrt{99}$

2. Simplify: $\sqrt{\frac{1}{36}}$, $\sqrt{\frac{144}{49}}$, $\sqrt{\frac{9}{4}}$, $\sqrt{\frac{36}{64}}$

3. Simplify:

$\sqrt{x^8y^2}$ $\qquad$ $\sqrt{81x^4y^{12}}$

$\sqrt{36x^6y^{14}}$ $\qquad$ $\sqrt{100x^2y^{10}z^8}$

SUMMARY:

1. Which answer exactly equals 12, 3.4641016 or $2\sqrt{3}$?
2. What is the difference between finding a square root and simplifying a radical?

HOMEWORK:

1. Simplify the expression:

$$\frac{2\sqrt{63}}{\sqrt{180}} \qquad \frac{\sqrt{72}}{\sqrt{24}} \qquad \frac{\sqrt{48}}{\sqrt{176}} \qquad \frac{2\sqrt{8}}{\frac{1}{2}\sqrt{200}}$$

2. Which of the expressions is $8\sqrt{5}$ equivalent to?

a) $\sqrt{64}$ b) $\sqrt{25}$ c) $\sqrt{320}$ d) $\sqrt{40}$

3. Which of the expressions is equivalent to $4\sqrt{19}$?

a) $\sqrt{126}$ b) $\sqrt{304}$ c) $\sqrt{216}$ d) $\sqrt{24}$

4. Simplify: $\sqrt{\frac{49}{144}}$, $\sqrt{\frac{36}{16}}$, $\sqrt{\frac{169}{25}}$, $\sqrt{\frac{196}{4}}$

5. Simplify: $\sqrt{64a^4b^6c^8}$, $\sqrt{36x^2y^4z^{10}}$, $\sqrt{49y^6x^8}$

6. Compute to the nearest hundredth.

a) $\sqrt{8}$ and $2\sqrt{2}$
b) $\sqrt{12}$ and $2\sqrt{3}$
c) $\sqrt{10}$ and $10\sqrt{.1}$

7. Compute $\sqrt{a^2+b^2}$ if

a) $a = 3$ and $b = 4$ c) $a = 7$ and $b = 24$
b) $a = 5$ and $b = 12$ d) $a = 9$ and $b = 40$

8. Compute $\sqrt{c^2}-\sqrt{a^2}$ if

a) $c = 8$ and $a = 4$ c) $c = 9\frac{1}{2}$ and $a = 6\frac{1}{2}$
b) $c = 13$ and $a = 11$ d) $c = 24\frac{1}{2}$ and $a = 23\frac{1}{2}$

9. Simplify assuming that no variable takes on a negative value.

a) $\sqrt{25a^2b^4c^6}$ d) $\sqrt{200a^5b^6c^7}$
b) $\sqrt{121r^6s^8t^{10}}$ e) $\sqrt{361x^2y^2z^2}$
c) $\sqrt{\frac{9u^6s^4t}{16u^2s^8t}}$ f) $\sqrt{\frac{1728s^6}{24t^4}}$

10. Simplify:

a) $\sqrt{100}$ b) $\sqrt{96}$ c) $\sqrt{125}$

d) $\sqrt{128}$ e) $\sqrt{1280}$ f) $\sqrt{760}$

11. Simplifying may be done in smaller steps.

For example: $\sqrt{128} = \sqrt{4 \cdot 32} = \sqrt{4}\sqrt{32} = 2\sqrt{32}$

$= 2\sqrt{4 \cdot 8} = 2\sqrt{4}\sqrt{8} = 2 \cdot 2\sqrt{8} = 4\sqrt{8}$

$= 4\sqrt{4 \cdot 2} = 4\sqrt{4}\sqrt{2} = 4 \cdot 2\sqrt{2} = 8\sqrt{2}$

More simply, $128 = \sqrt{64 \cdot 2} = 8\sqrt{2}$ (same as above)

Simplify each of the following using small steps:

a) $\sqrt{72}$ b) $\sqrt{288}$ c) $\sqrt{108}$

12. Simplify each of the following using "big" steps.

a) $\sqrt{27}$ b) $\sqrt{75}$ c) $\sqrt{800}$

13. Using the information that

$\sqrt{2} = 1.414$, $\sqrt{3} = 1.732$, $\sqrt{5} = 2.236$

estimate each of the following by first simplifying.

a) $\sqrt{32}$ b) $\sqrt{12}$ c) $\sqrt{45}$ d) $\sqrt{10}$

Lesson 29 - Adding and Subtracting Radicals

AIM: How do we add and subtract radicals?

PERFORMANCE OBJECTIVES: The student will be able to...

1. Combine radicals with like radicals.
2. Simplify unlike radicals by first simplifying each radical and then combine the like radicals.

VOCABULARY: index, radicand, like radicals, unlike radicals, like square root radicals

MOTIVATION:

1. Add like monomials: $6x + 5x$
2. Explain the rule you used.

DEVELOPMENT:

1. Discuss motivation. For the solution, we simplify by using the distributive property:
   $$6x + 5x = (6 + 5)x = 11x$$
   Recall the definition: Like monomials are monomials which have the same literal factor.
2. Referring to the motivation, change the variable to a fraction ($x = \frac{1}{19}$). The problem becomes $\frac{6}{19} + \frac{5}{19} = \frac{11}{19}$.
   Recall the definition: Like fractions are fractions which have the same denominator.
3. Referring to the motivation, change the variable to a radical ($x = \sqrt{2}$). The problem becomes $6\sqrt{2} + 5\sqrt{2} = 11\sqrt{2}$.
   Define: Like radicals are radicals which have the same index and the same radicand.
   Using the distributive property: $6\sqrt{2} + 5\sqrt{2} = (6 + 5)\sqrt{2}$
   $= 11\sqrt{2}$
   Note: You can add or subtract radicals in the same way you add or subtract like terms.
4. Do Application 1.

5. Consider the problem $\sqrt{2} + \sqrt{8}$.

We cannot use the distributive property as we did before, because these radicals are not like radicals. Ask: "How do we handle the problem of adding fractions which are not like fractions?" (By expressing each fraction as an equivalent fraction with a new denominator. When both fractions are expressed as equivalents with the same denominator, we can use the rule to find their sum.) Perhaps writing these two radicals in simpler form may provide a means of adding them.

$\sqrt{8} = \sqrt{4}\sqrt{2}$ or $2\sqrt{2}$

Thus: $\sqrt{2} + \sqrt{8} = \sqrt{2} + 2\sqrt{2} = (1 + 2)\sqrt{2} = 3\sqrt{2}$

(similar to $x + 2x = 3x$)

6. Consider: $4\sqrt{2} + 5\sqrt{8}$

$$4\sqrt{2} + 5\sqrt{4 \cdot 2} = 4\sqrt{2} + 5 \cdot 2\sqrt{2} = 4\sqrt{2} + 10\sqrt{2}$$
$$= (4 + 10)\sqrt{2} = 14\sqrt{2}$$

Consider: $3\sqrt{5x} - \sqrt{20x} = 3\sqrt{5x} - \sqrt{4 \cdot 5x} = 3\sqrt{5x} - 2\sqrt{5x} =$
$(3 - 2)\sqrt{5x} = \sqrt{5x}$

Rule: To combine unlike radicals:

a) Simplify each radical.

b) Combine like radicals by using the distributive property.

c) Compute the sum or difference.

7. Do Application 2.

8. Consider the following example:

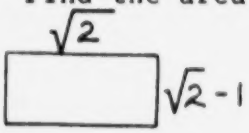
$$4\sqrt{6} + 3\sqrt{7} - 2\sqrt{6} = 4\sqrt{6} - 2\sqrt{6} + 3\sqrt{7}$$
$$= (4 - 2)\ 6 + 3\sqrt{7}$$
$$= 2\sqrt{6} + 3\sqrt{7}$$

By using commutative and distributive properties, combine like radicals.

Remember: $\sqrt{6}$ and $\sqrt{7}$ cannot be simplified and are not like radicals. Thus $2\sqrt{6} + 3\sqrt{7}$ cannot be expressed as a single term.

9. Do Application 3.

10. Consider: Find the area of the figure to the nearest tenth.



Area = length x width

$\sqrt{2}(\sqrt{2} - 1) =$

Use the distributive property to simplify.

$\sqrt{2}(\sqrt{2} - 1) = (\sqrt{2})(\sqrt{2}) - (\sqrt{2})(1) = 2 - \sqrt{2} \approx$

$2 - 1.41 = .59 \approx .6$ Answer

11. Do Applications 4, 5, 6.

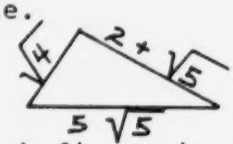
APPLICATIONS:

1. Simplify:
   a) $5\sqrt{2} + 2\sqrt{2}$
   b) $9\sqrt{2} - \sqrt{2}$
   c) $8\sqrt{3x} - 3\sqrt{3x}$
   d) $5\sqrt{7} - 4\sqrt{7}$
   e) $4\sqrt{3} + 2\sqrt{3} - 5\sqrt{3}$
   f) $\sqrt{x} + 5\sqrt{x}$

2. Simplify:
   a) $7\sqrt{3} - 4\sqrt{12}$
   b) $7\sqrt{3x} - \sqrt{12x}$
   c) $7x\sqrt{5xy^2} - \sqrt{20x^3y^4}$
   d) $\sqrt{75} - \sqrt{27} + \sqrt{12}$
   e) $\sqrt{8y^3} - \sqrt{2y^3}$

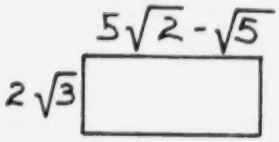
3. Simplify, if possible:
   a) $4\sqrt{3} + \sqrt{2} - 4\sqrt{2}$
   b) $\sqrt{22} + \sqrt{11} - \sqrt{66}$

4. Find the perimeter of the triangle.

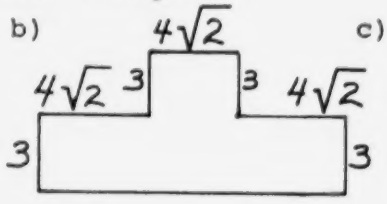


5. Find the perimeter and area of each figure in radical form and to the nearest integer.

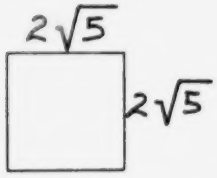
a)



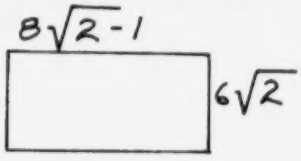
b)



c)



6. Find the area in radical form.

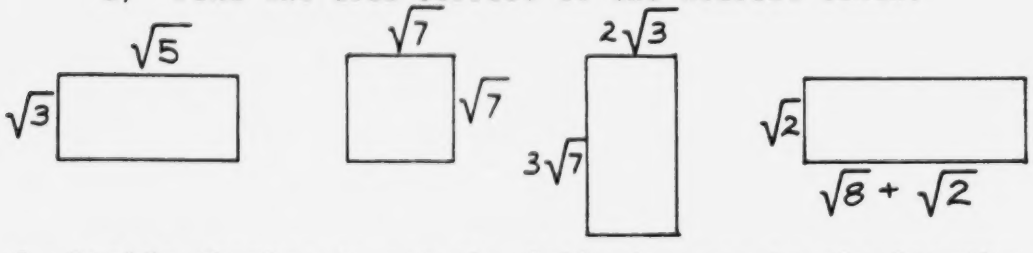


SUMMARY:

1. Does $\sqrt{a+b} = \sqrt{a} + \sqrt{b}$ for any nonnegative value given to a and b?
2. How do we combine like radicals?
3. How do we combine unlike radicals?

HOMEWORK:

1. Find the sum of:
   a) $\sqrt{2} + 3\sqrt{2}$ b) $\sqrt{18} + 15\sqrt{2}$
2. Simplify:
   a) $\sqrt{a} + 8\sqrt{a}$ d) $\sqrt{18} - 2\sqrt{2}$
   b) $5\sqrt{3} + \sqrt{12}$ e) $4\sqrt{5} + 7\sqrt{2} - 7\sqrt{5}$
   c) $\sqrt{72x^3} - \sqrt{18x^3}$ f) $\sqrt{50} + 3\sqrt{2}$
3. Find the value of the expression $\sqrt{18} - \sqrt{2}$.
4. a) Find the perimeter of the figures in radical form.
   b) Approximate the perimeter correct to the nearest tenth.
   c) Find the area in radical form.
   d) Find the area correct to the nearest tenth.



5. Decide whether or not the following can be the lengths of the sides of a triangle. Remember that the longest side must be less than the sum of the other two sides.
   a) $\sqrt{3}, 2\sqrt{7}, \sqrt{12}$ e) $\sqrt{3}, \sqrt{3}, \sqrt{13}$
   b) $\sqrt{3}, \sqrt{5}, \sqrt{11}$ f) $\sqrt{2}, \sqrt{3}, \sqrt{8}$
   c) $\sqrt{24}, \sqrt{12}, \sqrt{12}$ g) $\sqrt{6}, \sqrt{7}, \sqrt{29}$
   d) $\sqrt{.24}, \sqrt{.12}, \sqrt{.12}$ h) $\sqrt{10}, \sqrt{15}, \sqrt{40}$
6. State TRUE or FALSE. If FALSE, give an example to show that it is FALSE.
   a) If $n > 0$ then $\sqrt{n} \geq n$.
   b) If $a > 0$ and $b > 0$, then $\sqrt{a+b} = \sqrt{a} + \sqrt{b}$.

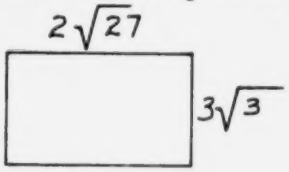
c) If $a > 0$ and $b > 0$, then $\sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}$.

d) If $a > b > 0$, then $\sqrt{a-b} > \sqrt{a} - \sqrt{b}$.

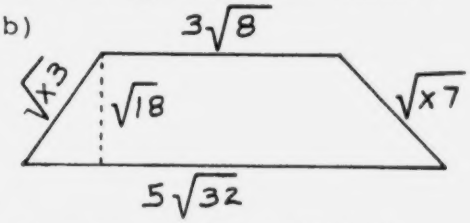
e) If $0 < a < b$ and $a < b - a$, then $\sqrt{a} < \sqrt{b} - \sqrt{a}$.

7. Find the area and perimeter and simplify your answer:

a)



b)



8. Solve completely:

a) $x^2 = 25$

b) $x^2 = 24$

c) $(x - 7)(2x + 5) = 0$

d) $(x - \sqrt{3})(x + \sqrt{3}) = 0$

9. Multiply $\sqrt{2}(\sqrt{18} - 3\sqrt{2})$ and simplify.

Lesson 30 - Pythagorean Theorem

AIM: What is the Pythagorean theorem?

PERFORMANCE OBJECTIVES: The student will be able to...

1. State the Pythagorean theorem (formula).
2. Identify the legs and hypotenuse of a right triangle.
3. Solve numeric and algebraic problems involving the Pythagorean theorem leading to rational and irrational answers.
4. Identify Pythagorean triples.
5. Determine whether a triangle is a right triangle, given the length of its sides.

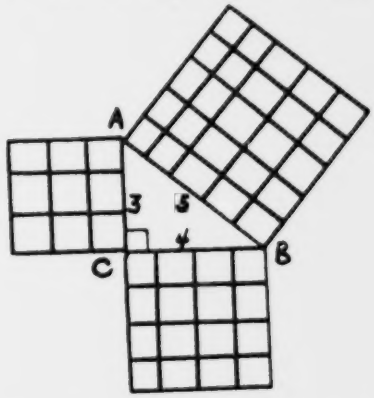
VOCABULARY: right triangle, legs, hypotenuse, diagonal of rectangle, diagonal of square, Pythagorean

MOTIVATION:

Given right triangle ABC and the three squares shown:

a) Find the area of each square.

b) What relationships do you observe?

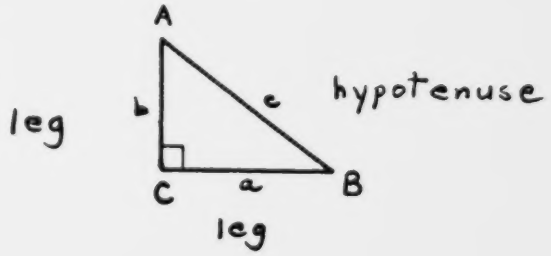


DEVELOPMENT:

1. Refer to motivation. Identify from the diagram the legs and hypotenuse of a right triangle.

RIGHT TRIANGLE ABC

Review:



Elicit from the class:

Area of Square I = 3·3 = 9
Area of Square II = 4·4 = 16
Area of Square III = 5·5 = 25

Area of Square III = Area of Square I + Area of Square II

$25 = 9 + 16$

$c^2 = a^2 + b^2$

In this triangle, the sum of the areas of the small squares equals the area of the large square (or the sum of the squares of the lengths of the short sides equals the square of the length of the longest side).

Ask, "Do you think that this relationship will hold for other triangles?" Students may suggest that the statement will be true for:

a) all right triangles
b) all triangles whose sides are consecutive integers
c) all triangles

It may quickly be seen that the relation does not hold for cases b and c (e.g., $4^2 + 5^2 \neq 6^2$); students should discover that the relation seems to hold for all right triangles.

Ask the students to draw several right triangles, label the hypotenuse c and the legs a and b, measure these sides and fill in a chart similar to the one below.

TRIANGLE	a	b	c	a^2	b^2	$a^2 + b^2$	c^2

The student should see that $a^2 + b^2 = c^2$ for every right triangle.

Elicit in words: It appears to be true that:

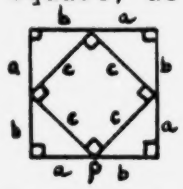
In a right triangle, the square of the length of the hypotenuse is equal to the sum of the squares of the lengths of the legs.

This relationship is known as the Pythagorean theorem, in honor of Pythagoras, a Greek mathematician, who proved the relationship in 500 B.C.

Pythagoras' "proof" was probably a simple demonstration such as the one following.

Place four congruent right triangles with each hypotenuse

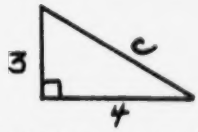
along the side of a square, as shown:



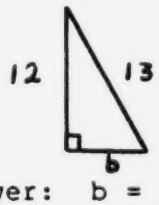
Ask students to explain why a straight angle is formed at point P (the acute angles of a right angle are complementary.) What type of figure is the large quadrilateral? (A square; each side has length a+b.)
The area of each triangle = $\frac{1}{2}ab$.
Area of larger square = Area of smaller square plus the Areas of four right triangles.

$$(a + b)^2 = c^2 + 4(\tfrac{1}{2}ab)$$
$$a^2 + 2ab + b^2 = c^2 + 2ab$$
$$a^2 + b^2 = c^2$$

3. The Pythagorean formula may be used to find the length of a side of a right triangle when two sides are known, as in the following examples.

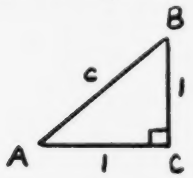


Answer: c = 5



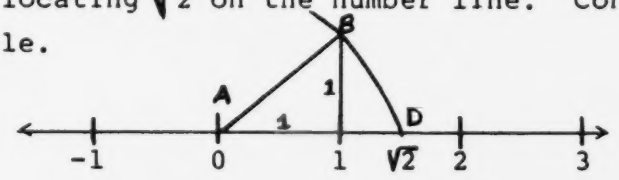
Answer: b = 5

4. Consider: Two roads, each a mile long, intersect at right angles. A third road will connect directly the ends of the other two roads. How long will the third road be?
Draw diagram:



$$c^2 = a^2 + b^2$$
$$c^2 = 1^2 + 1^2$$
$$c^2 = 2$$
$$c = \pm\sqrt{2}$$ (eliminate negative value)
$$c = \sqrt{2}$$

$\sqrt{2}$ is an irrational number.
Demonstrate locating $\sqrt{2}$ on the number line. Construct a right triangle.



Since AB = $\sqrt{2}$, we use length AB as a radius and A as the center to find the point D; then $\overline{AD}$ has the length $\sqrt{2}$ and D corresponds to the point which is at a distance of $\sqrt{2}$ from zero.

Similarly many irrational numbers can be graphed on the number line.

5. Do Application 1.

6. Use the Pythagorean theorem to name several Pythagorean triples (lengths that may be measures of the sides of a right triangle): (3,4,5) (5,12,13) (8,15,17). Since triangles similar to these will also be right triangles, new triples may be formed by taking <u>multiples</u> of these lengths.

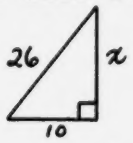
e.g.,

6,8,10	10,24,26	16,30,34
9,12,15	15,36,39	24,45,51
12,16,20	20,48,52	32,60,68

7. Do Application 2.

8. Help students acquire skill in solving problems.

   a) A 26 foot ladder is placed against a wall with the foot resting 10 feet from the wall. How high up the wall does it reach?



   Refer to Pythagorean triple 5,12,13 and its multiple 10,24,26. Therefore x = 24.

   b) Suppose a baseball diamond is actually a square with sides of 90 feet. To the nearest foot, how far is home plate from second base? Answer: $90\sqrt{2}$

<u>Note</u>: By varying the size of the square and using $\sqrt{ab} = \sqrt{a}\cdot\sqrt{b}$ to simplify the results, we can generalize that the diagonal of a square with side S is equal to $S\sqrt{2}$.

9. Do Applications 3-7.

10. The converse of the Pythagorean theorem is also true. To form the converse of a conditional statement, reverse the <u>if</u> and <u>then</u> parts.

If the square of one side of a triangle is equal to the sum of the squares of the lengths of the other two sides, the triangle is a right triangle. The longest side is the hypotenuse.

If the lengths of the sides of a triangle are 4, 5 and 6, is the triangle a right triangle?

Review that the hypotenuse is the longest side in a right triangle. Therefore c, the length of the hypotenuse, must be the largest number. The remaining numbers may be substituted for either a or b.

$$c^2 = a^2 + b^2$$
$$6^2 \stackrel{?}{=} 4^2 + 5^2$$
$$36 \stackrel{?}{=} 16 + 25$$
$$36 \neq 41$$

The triangle is not a right triangle since the conditions of the Pythagorean theorem are <u>not</u> satisfied.

Consider the sides of a triangle to be 6, 8 and 10. Do these sides form a right triangle?

$$c^2 = a^2 + b^2$$
$$10^2 \stackrel{?}{=} 6^2 + 8^2$$
$$100 \stackrel{?}{=} 36 + 64$$
$$100 = 100$$

The triangle is a right triangle.

11. Do Application 8.

APPLICATIONS:

1. Find the missing measure in each right triangle.

   a = 2, b = 3, c = ?

   a = 5, b = ?, c = 6

   a = ?, b = 2, c = 4

2. Verify that each is a Pythagorean triple.

   3,4,5 5,12,13

   7,24,25 9,40,41

   11,60,61

   Can you see a pattern in the formation? List the next three triples which fit the pattern.

3. Find the length of the diagonal of a rectangle whose sides are 8 and 15; 1 and 2.

4. Find the length of the diagonal of a square whose side is $\sqrt{3}$.

5. Jeff walked 7 km north and 3 km west. How far was he from the starting point?

6. Find the length of the side of a square if the length of the diagonal is 4 cm.

7. If the length of the two longest sides of a right triangle are 3 and 4, find the length of the shortest side to the nearest tenth.

8. The measures of three sides of a triangle are given in each of the following. Determine whether each triangle is a right triangle.

   4,5,6 12,16,20 2, 3, 4

SUMMARY:

1. What relationship did we discover for the sides of a right triangle?

2. How do you use the Pythagorean theorem to determine whether or not a triangle is a right triangle?

3. How do we find the diagonal of a square?

4. Write the Pythagorean theorem and its converse as a biconditional.

5. The perimeter of a right triangle is 30 inches. If the hypotenuse is 13, find the length of each leg.

HOMEWORK:

1. A movie screen is 12 ft. by 10 ft. What is its diagonal length?

2. The legs of a right triangle have lengths of 2 and 7. Express, in radical form, the length of the hypotenuse.

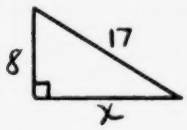
3. Find the length of the side of a square which has a diagonal measuring $6\sqrt{2}$ in.

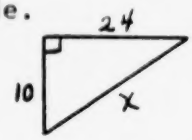
4. The length of one leg of a right triangle is 8. The lengths of the other leg and the hypotenuse are consecutive odd integers. Find the length of the hypotenuse.

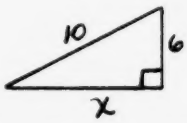
5. Juliana walked 7 km north and 3 km east. How far was she from her starting point to the nearest tenth?

6. A 4 m ladder is 1 m from the base of a building. At what height does it touch the building?

7. Find the missing side.



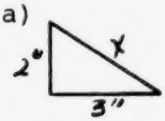




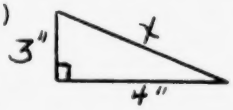
8. If the lengths of the sides of a triangle are 9, 40 and 41, is the triangle a right triangle?

9. Compute the length of the missing side to the nearest tenth of an inch and check with a scale drawing.

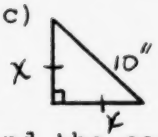
a)



b)



c)



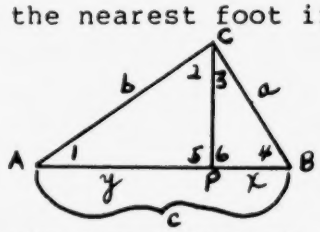
d)

square

Area of square = 100

10. Find the perimeter of a square to the nearest foot if its area is 100 square feet.

*11. In right triangle ABC,
∠C is a right angle.
$\overline{CP}$ is drawn perpendicular to $\overline{AB}$.



For each statement below, explain why the statement is true.

a) $\Delta ACP \sim \Delta ABC$

b) $\Delta CBP \sim \Delta ABC$

c) $\frac{y}{b} = \frac{b}{c}$

d) $\frac{x}{a} = \frac{a}{c}$

e) $b^2 = yc$

f) $a^2 = xc$

g) $a^2 + b^2 = xc + yc$

h) $a^2 + b^2 = c(x + y)$

i) $a^2 + b^2 = c^2$

*12. If the perimeter of an isosceles right triangle is 20 inches, find the length of a leg and the hypotenuse to the nearest tenth of an inch.

Lesson 31 - Multiplying Monomials

AIM: How do we multiply a monomial by a monomial?

PERFORMANCE OBJECTIVES: The student will be able to...

1. Recall the procedure for multiplying monomials.
2. Multiply monomials with the same base and literal exponents. e.g., $2^a \cdot 2^b = 2^{a+b}$

VOCABULARY: exponent, base, coefficient, monomial

MOTIVATION:

If x is an integer, which of the following statements is true?

a) $2^x < x^2$
b) $2^x > x^2$
c) $2^x = x^2$
d) The relationship cannot be determined without further information about x.

DEVELOPMENT:

1. Discuss motivation problem. In x^2, 2 is the exponent, x is the base. The exponent tells us the number of times the base is to be used as a factor. $5^2 = 5 \cdot 5 = 25$; $2^5 = 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 = 32$. When $x = 5$, $x^2 > 2^x$. Will this always be the case? Elicit values for x for which $x^2 > 2^x$, e.g., $3^2 = 9 > 8 = 2^3$. Are there any values for x which give the expressions x^2 and 2^x the same value? What values do the expressions have when $x = 0$? ...when x is negative? Students should recall that, by definition, $x^0 = 1$ and $x^{-n} = \frac{1}{x}n$. Therefore $2^{-3} = \frac{1}{2}3 = \frac{1}{8}$; when $x = -3$, $x^2 = (-3)^2 = +9$. Elicit that a good perspective on the problems may be obtained by organizing values of x, 2^x and x^2 in a chart.

x	2^x		x^2
-3	$\frac{1}{8}$	<	+9
-2	$\frac{1}{4}$	<	+4
-1	$\frac{1}{2}$	<	+1
0	1	>	0
1	2	>	1
2	4	=	4
3	8	<	9
4	16	=	16
5	32	>	25
6	64	>	36
7	128	>	49
8	256	>	64

The answer to the problem is choice D, since the relation depends on x as follows:

$2^x < x^2$ when x is negative or x = 3

$2^x = x^2$ when x = 2 or x = 4

$2^x > x^2$ for all other integer values of x

2. Recall the procedure for multiplying powers of the same base. $x^4 \cdot x^3 = (x \cdot x \cdot x \cdot x)(x \cdot x \cdot x) = x^7$. The general rule is to keep the base and add the exponents of the factors: $x^2 \cdot x^3 = x^5$; $x^a \cdot x^b = x^{a+b}$

3. Consider literal exponents. Ask: "What is the value of $2^x \cdot 2^5$?" $2^x \cdot 2^5 = 2^{x+5}$. (Verify by using various values for x that this is correct and 2^{5x} is not correct.) More generally, $2^x \cdot 2^y = 2^{x+y}$.

4. Recall the rule for finding a power of a power: $(x^4)^3 = (x^4)(x^4)(x^4) = x^{4+4+4} = x^{12}$. In general, $(x^a)^c = x^{ac}$.

5. Do Applications 1 and 2.

6. Recall: To find the product of two monomials
   a) we multiply the numerical coefficients, then
   b) we multiply the variable factors that are powers having the same base.

   e.g.,

   $(4x^2)(3x^4) = 12x^6$

   $(5x)(3y) = 15xy$

   $(2x^2)(4x^5) = 8x^7$

   $(2a^2b^4)(-5a^3b) = -10a^5b^5$

   Have students examine $4x^2$, $3x^4$ and $12x^6$. Are they monomials?

Have students discover that multiplying a monomial by a monomial results in a monomial.

7. Consider $(x^2)(3x^2)(\frac{1}{3}x^3)(-4) = (3)(\frac{1}{3})(-4)(x^2)(x^2)(x^3) = -4x^7$. Have students discover that the result of multiplying several monomials is also a monomial.

8. Do Application 3.

9. Consider the following expressions: $3x^2$, $(3x)^2$. Are they equivalent?

   $3x^2 = 3 \cdot x \cdot x = 3x^2$

   $(3x)^2 = 3x \cdot 3x = 9x^2$

   Have students discover that in raising a product to a power, each of the factors of the product is raised to that power.

10. Do Application 4.

11. We can apply monomial multiplication to area. If the area of a rectangle equals length times width, find the area of a rectangle whose length equals 4y and whose width equals 2y. $A = l \cdot w = 8y^2$

    Do Application 5.

APPLICATIONS:

1. Compute the products or powers.

$x^6 \cdot x^3$	$5^4 \cdot 5^3$
$y^5 \cdot y$	$a^4 \cdot a^6$
$(z^2)^5$	$r^3 \cdot r^3$
$(ab^2)^3$	$t^7 \cdot t^4 \cdot t^2$

2.

$y^2 \cdot y^{2a}$	$5^c \cdot 5^2$
$x^m \cdot x$	$x^{4a} \cdot x^{2a}$
$5^x \cdot 5^y$	

3.

$(7xy)(3x)$	$(-3x^2)(-5x^4)$
$(6r^4s)(-4rs)$	$(-7x^2y^4)(-2xy^2)$
$(-5a^4)(a)$	$(7xy)(-2ab)$

$(3a^5b^4)(6a^2b^7)$ $(-4x^2)^3$

$(-7c)(8cd^2)$ $(-2x^2y^3)(8x^4y^2)$

$(3x)(4x^2y)(-8x^3y^2)$

The product of $(-7a^3)$ and $(2a^2)$ is

a) $-14a^6$ b) $9a^6$ c) $-9a^5$ d) $-14a^5$

4. Compute: $(4x)^2$; $(-3y)^4$

5. a) Find the area of a rectangle whose length is $4cd^2$ and width is 2cd.
   b) Find the volume of a cube whose side is 4x.
   c) Solve for x: $5^x \cdot 5^{10} = 5^{15}$

SUMMARY

1. Name the coefficient, base and exponent in $4x^5$.
2. Explain the difference between $x^3 \cdot x^6$ and $(x^3)^6$.
3. Simplify $(3x^3)^2$ and explain how the answer was obtained.

HOMEWORK

1. Compute each product:

$(8x^4)(3x)$ $(-4a^3)(+4a^3)$

$(-8r)(3st)$ $(-6a^3b)(5a^2b^2)$

$(-4x^5)(-7x^4)$ $3^{2x} \cdot 3^{4x}$

$(-2x^2y^2)$ $(-3y^2)(-y)$

$(7xy)(2x^2y^3)(4xy^3z)$ $(3cd^2)(-8cd)$

$x^{5a} \cdot x^b$

2. When $5xy^2$ is multiplied by $4x^3y^5$, the product is:

a) $9x^4y^7$ b) $20x^4y^7$ c) $20x^3y^{10}$ d) $9x^3y^{10}$

3. Find the area of a rectangle whose length is 6y and width is 3y.

   Find the area of a square whose side is 6x.

   Find the volume of a cube whose side is 9a.

4. Compute the square of each of the following.

$4a$ $5ab$ $-3a^{2x}$

5. Solve for x.

$2^x \cdot 2^x = 2^{16}$

$2^{10} = 2^x$

$3^x = 27$

6. Compute:

a) $(-3x^3)^2$

b) $(-3x^2)^3$

c) $2^5 - 5^2$

d) $(2.5ab^3)(5 \cdot 2a^3b)$

e) $(2\frac{1}{3})y^2z(9xyz)$

f) $(3\frac{1}{7}r^2)(42)$

7. State TRUE or FALSE. If FALSE, show by example.

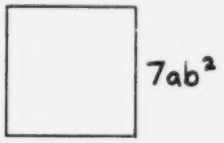
a) If $a > b > 0$, $a^b > b^a$.

b) If $a > b > 0$, $a^b \neq b^a$

c) If $a > b > 0$, then $a^2 > b^2$ and $a^3 > b^3$.

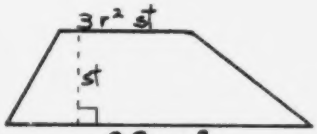
8. Compute the area of each region in terms of the given variables.

a)



square

b)



trapezoid

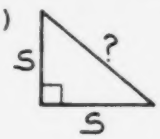
9. A product may be checked by replacing the variable with specific numbers. Compute each product and check:

a) $(3xy)(2xy^2)$

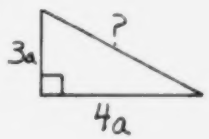
b) $(6rs)^2$

10. Express the length of the missing side in terms of the given variables.

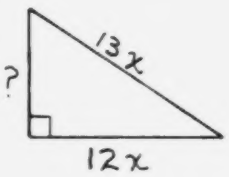
a)



b)



c)



Lesson 32 - Multiplying a Polynomial by a Monomial

AIM: How do we multiply a polynomial by a monomial?

PERFORMANCE OBJECTIVES: The student will be able to...

1. Recall the use of the distributive property.
2. Multiply a polynomial by a monomial.

VOCABULARY: distributive property

MOTIVATION:

Draw a rectangle whose length is $3x + 4$ and whose width is x. Write an expression in terms of x to represent the area. Verify that the expression you have written is correct by substituting these values for x: 10,7,23.

DEVELOPMENT:

1. We know the distributive property states $a(b + c) = ab + ac$. Therefore: $x(3x + 4) = x(3x) + x(4)$
$= 3x^2 + 4x$

This may be illustrated by separating the rectangle into two rectangles so that the length of one rectangle equals $3x$ and the length of the other one equals 4. The polynomial $3x^2 + 4x$ is called the <u>product</u> of the monomial x and the polynomial $3x + 4$.

Here are some additional examples to show how to multiply a polynomial by a monomial. Note that the distributive property is used in each example.

$5(3x+2y) = 5(3x) + 5(2y) = 15x + 10y$

$4n(n^2+3n+2) = 4n(n^2) + 4n(3n) + 4n(2) = 4n^3 + 12n^2 + 8n$

$3y^2(2y^3-5y+4) = 3y^2(2y^3)+(3y^2)(-5y)+3y^2(4)$
$= 6y^5 - 15y^3 + 12y^2$

Remember: To multiply a polynomial by a monomial we use the distributive property of multiplication over addition. Each term within the parentheses must be multiplied by the monomial preceding the parentheses. The multiplication may also be arranged vertically.

$$\begin{array}{r} 5x - 4y \\ \underline{\qquad 3x} \\ 15x^2 - 12yx \end{array}$$

2. Do Application 1.

3. If the monomial is negative, be careful. All of the signs of the polynomial will change:

   $-3(x - 8) = -3x + 24$

4. Do Application 2.

5. The distributive property could also be written as $(b + c)a = ba + ca$.

   For example, $(3x + 7)5x = 15x + 35x$

   We can use our new knowledge of polynomials to solve new kinds of equations:

$$\begin{aligned} x(x + 2) - x(x - 4) - 3x &= 21 \\ x^2 + 2x - x^2 + 4x - 3x &= 21 \\ 6x - 3x &= 21 \\ 3x &= 21 \\ x &= 7 \end{aligned}$$

6. Do Application 3.

APPLICATIONS:

1. a) Compute the indicated products:

   $5x^2(9x - 2xy)$ $\qquad$ $3(5a + 6d)$

   $6x(8x + 3)$ $\qquad$ $7x(x - 9)$

   b) Compute the product: $6a^2b(5a - 2ab + a^2b)$.

   c) Simplify: $ab(a + b)$.

   d) Remove parentheses: $5d(d^3 + 4d)$.

   e) Simplify: $5x^2y(-2x^3 + 4xy - 9x^2)$

   f) Find the area of a rectangle with a length of $5x$ and a width of $(2x^2 + 4x - 3)$.

2. Compute:

   $-3(3t - 4s)$ $\qquad$ $-4e(5e - e^2)$

   $-8(5x^2 - 4x + 2)$ $\qquad$ $-xy(5x^4y - 2x^3)$

   $-5a^2(6a - 2a^2)$

3. Solve for x: $2x - 1 - 4(x + 2) = 11$
   Solve for y: $3(7 - 2y) - 4(8 + 3y) = 25$

SUMMARY:

1. Find the error: $-8(x + 2) = -8x + 16$
2. What is the procedure for multiplying a polynomial by a monomial?
3. Find the perimeter of a square if each side is $3x + 7$.

HOMEWORK:

1. Compute the indicated products:

$4(x^2 + 5x)$	$3(4x + 2)$
$5x(x^2 + 2x - 1)$	$6(2x^3 + x^2 - 5)$
$8x(4x - 2)$	$-9(4x + 8)$
$ab(3a^2b + 2ab^2 + 4a^3)$	$2x^2(4x^3 - 9x^2 + 7x + 2)$
$-5x(x^2 + 6x - 4)$	$8(6x - 2y + 5)$

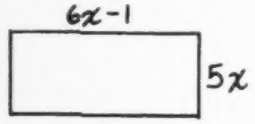
2. Compute the area of a rectangle with the dimensions $6x$ and $4x^2 + 2x - 5$.
3. Simplify: $8x^2y^2(-4x^2 + 5xy - 9x^2y)$
4. Solve for a: $6(a^2 - 5a) - 2a(3a + 3) = 72$
5. Compute the products using distributivity.

a) $8(40 + 2)$	c) $8(80 - 1)$
b) $7(60 + 7)$	d) $9(100 - 1)$

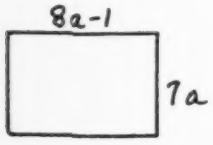
6. Compute:

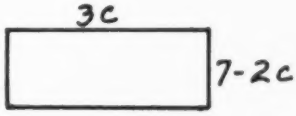
a) $4a(a^3 + a - 1)$	d) $3ab^2(a^2b - ab^2)$
b) $-3b(2b^2 - b - 3)$	e) $-7ac^2(-2c + 3a)$
c) $5c^3(-3c + 7)$	f) $a(b + c + d)$

7. Compute the area of each rectangle.

a) 

c)

b) 

d) 

8. Products may be checked by assigning numerical values to each variable and observing the values of the products with and without distributivity. Compute each product and check by assigning numerical values:

   a) $3x(x + 3)$
   b) $5y(3y - 2)$
   c) $-2a(a^2 - 3a + 1)$
   d) $7c(2 - c - 3c^2)$

9. Solve and check.

   a) $3(2x^2 + x - 7) = 2x(3x + 1)$
   b) $-7y(y + 3) = y(6 - 7y) + 27$

10. Compute to the nearest tenth:

   a) $\sqrt{.9}$
   b) $\sqrt{90}$
   c) $\sqrt{.09}$

Lesson 33 - Multiplying Two Binomials

AIM: How do we compute the product of two binomials?

PERFORMANCE OBJECTIVES: The student will be able to...

1. Write the product of two binomials with at most two variables.
2. Form the product by multiplying either
   a) horizontally
   b) vertically

VOCABULARY: review: binomial, trinomial, product, horizontal, vertical

MOTIVATION:

The length of a rectangle is x + 5 and the width is x + 2. Express the area in terms of x.

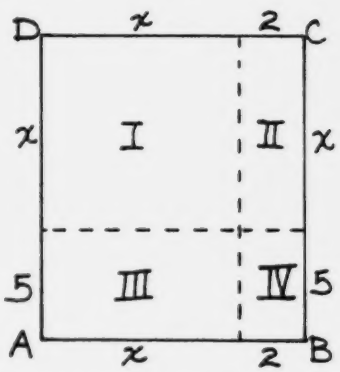
DEVELOPMENT:

1. Refer to motivation. Using the formula, we write:

   Area = (length)(width)

   $A = (x + 5)(x + 2)$

   Method 1: Demonstrate geometrically the product $(x + 5)(x + 2)$. Have students examine the diagram. The area of the larger rectangle ABCD equals the sum of the areas of quadrilaterals I, II, III and IV.



   Consider the smaller quadrilaterals separately and find the area of each.

   Area of I = $(x)(x) = x^2$

   Area of II = $(2)(x) = 2x$

   Area of III = $(5)(x) = 5x$

   Area of IV = $(5)(2) = 10$

Area of ABCD = Area of I + Area of II + Area of III + Area of IV

$= x^2 + 2x + 5x + 10$

$= x^2 + 7x + 10$

Method II: Have students multiply $(x + 5)(x + 2)$ using the associative and distributive properties.

$(x + 5)(x + 2) = (x + 5)(x) + (x + 5)(+2)$ Why?

$= x^2 + 5x + 2x + 10$ Why?

$= x^2 + (5x + 2x) + 10$ Why?

$= x^2 + (5 + 2)x + 10$ Why?

$= x^2 + 7x + 10$

Have students notice that the distributive property of multiplication over addition is used several times. Compare the answers in Method I and Method II. Consider the product of $(x + 2)(x + 5)$, will it yield the same result as above? Why?

2. Have students check the accuracy of the multiplication by substituting numerical value for x to determine whether or not a true statement results.

Let $x = 10$

$(x + 5)(x + 2)$ $(10 + 5) = 15$, $(10 + 2) = 12$

$(15)(12) = 180$

or

$(x + 5)(x + 2) =$

$(10 + 5)(10 + 2) = (10 + 5)(10) + (10 + 5)(2)$

$= 100 + 50 + 20 + 10$

$= 180$

3. Consider the product of $(3x + 2)(x + 5)$.

Using the distributive property:

$(3x + 2)(x + 5) = (3x + 2)(x) + (3x + 2)(5)$

$= 3x^2 + 2x + 15x + 10$

$= 3x^2 + (2 + 15)x + 10$

$= 3x^2 + 17x + 10$

Have students check the accuracy by substituting numerical values for x.

4. Do Application 1.

5. Have students learn to multiply the two binomials when they are in a vertical arrangement.

Remember $(x + 5)(x + 2) = x^2 + 7x + 10$

Have students let x equal 10.

If $x = 10$, $x + 5 = 15$, $x + 2 = 12$ and $x^2 + 7x + 10 = 180$.

	$15 =$	$10 + 5 =$	$10 + 5$	$x + 5$
	$\underline{12 =}$	$\underline{10 + 2}$	$\underline{10 + 2}$	$\underline{x + 2}$
2×15	30	$20 + 10$	$2(10) + 10$	$2x + 10$
10×15	$\underline{15}$	$\underline{100 + 50}$	$\underline{10^2 + 5(10)}$	$\underline{x^2 + 5x}$
	180	$100 + 70 + 10$	$10^2 + 7(10) + 10$	$x^2 + 7x + 10$

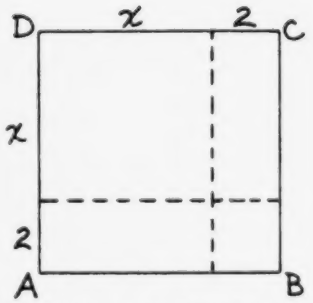
Multiply $(3x + 2)(x + 5)$ vertically

$$\begin{array}{r} 3x + 2 \\ \underline{x + 5} \\ 15x + 10 \\ \underline{3x^2 + 2x} \\ 3x^2 + 17x + 10 \end{array}$$

6. Do Application 2.

7. Find the area of a square with side $x + 2$.

Illustrate geometrically.



Area of I = $x \cdot x = x^2$

Area of II = $2 \cdot x = 2x$

Area of III = $2 \cdot x = 2x$

Area of IV = $2 \cdot 2 = 4$

Area of ABCD = Area of I + Area of II + Area of III + Area of IV

$= x^2 + 2x + 2x + 4$

$= x^2 + 4x + 4$

Area = (length)(width) = $(x + 2)(x + 2)$

Note: The product of two equal binomials is called a perfect square trinomial.

Horizontally:

$$\begin{aligned} (x + 2)(x + 2) &= (x + 2)x + x + 2)(2) \\ &= x^2 + 2x + 2x + 4 \\ &= x^2 + (2 + 2)x + 4 \\ &= x^2 + 4x + 4 \end{aligned}$$

Vertically:

$$\begin{array}{r} x + 2 \\ \underline{x + 2} \\ 2x + 4 \\ x^2 + 2x \\ \hline x^2 + 4x + 4 \end{array}$$

8. Do Application 3.

9. Have students learn to multiply the sum and difference of two terms.

   Consider the product of $x + 2$ and $x - 2$.

$$\begin{array}{r} x + 2 \\ \underline{x - 2} \\ -2x - 4 \\ x^2 + 2x \\ \hline x^2 \quad - 4 \end{array} \qquad \begin{array}{r} x + y \\ \underline{x - y} \\ -xy - y^2 \\ x^2 + xy \\ \hline x^2 \quad - y^2 \end{array} \qquad \begin{array}{r} 2x + 3 \\ \underline{2x - 3} \\ -6x - 9 \\ 4x^2 + 6x \\ \hline 4x^2 \quad - 9 \end{array}$$

   Why do the above products contain only two terms and not three?

   Rule: To multiply the sum of two terms by the difference of two terms:

   a) Square the first term.

   b) From the result subtract the square of the second term.

   $(a + b)(a - b) = a^2 - b^2$

10. Do Application 4.

APPLICATIONS:

1. Compute the products using horizontal form.

   $(x + 5)(x + 3)$ $\qquad$ $(x - 1)(x + 5)$

   $(y - 3)(2y + 5)$ $\qquad$ $(3a - 1)(4a - 5)$

2. Compute the products for the examples in 1. vertically.

3. Find the perfect square trinomial.

   $(x - 3)(x - 3)$ $\qquad$ $(y + 5)^2$

   $(a + 2b)^2$

4. Multiply:

   $(x + 1)(x - 1)$ $\qquad$ $(c + d)(c - d)$

   $(2x - 1)(2x + 1)$ $\qquad$ $(3x + 4)(3x - 4)$

SUMMARY:

1. Find an example when the product of two binomials is a trinomial.
2. Find an example when the product of two binomials is a binomial.
3. Find an example when the product of two binomials is a polynomial of four terms.

HOMEWORK:

1. Express the product $(x + 2)(x - 5)$ as a trinomial.
2. Express the product $(x - 3)(x + 3)$ as a binomial.
3. Express as a trinomial $(3x - 1)(x + 2)$.
4. Compute the products for the following using

a) horizontal form	b) vertical form
$(x - 2)(x + 6)$	$(3a + 2)(4a - 1)$
$(7a + 2)^2$	$(4x + y)(5x - 2y)$

5. Compute the products for the following using distributivity and check by giving the variables a numerical value:

a)	$(2a - 7)(3a + 4)$	e)	$(6g + 9h)(6g - 9h)$
b)	$(3b + 4)(2b - 5)$	f)	$(2\frac{1}{2} + 1)(2\frac{1}{2} - 1)$
c)	$(7c + 2a)(5c - 2a)$	g)	$(2.5r + 3s)(2.5r - 3s)$
d)	$(3d - 2e)(3d + 2e)$	h)	$(2x - 3y)(2x + 3y)$

6. Compute the products using the vertical format and check by giving the variables a numerical value:

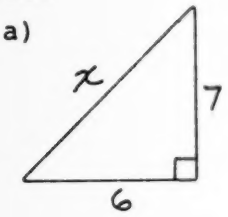
a)	$(3s + 7t)(2s - 3t)$	c)	$(2a + 3b)(2c + 3d)$
b)	$(5x + 2y)(5x - 3y)$	d)	$(6u - 7v)(6u + 7v)$

7. Show by means of areas of rectangles that: $(a + b)(a - b) = a^2 - b^2$. Assume $a > b$.
8. Solve:

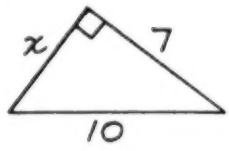
a)	$(x + 2)(x - 3) = 0$	e)	$y^2 - 2.25 = 0$
b)	$(2y - 3)(2y + 3) = 0$	f)	$9s^2 = 25$
c)	$(u + 1)(u - 1) = 0$	g)	$4t^2 = 81$
d)	$x^2 - 81 = 0$	h)	$.16a^2 - 1 = 0$

9. Compute to the nearest tenth the length of the missing side.

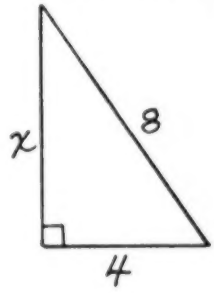
a)



b)



c)



Lesson 34 - Multiplying Two Binomials

AIM: How do we multiply two binomials mentally?

PERFORMANCE OBJECTIVES: The student will be able to...

1. Write the product of two binomials using the FOIL method.
2. Solve problems using multiplication of two binomials.

VOCABULARY: no new terms

MOTIVATION:

Find each product:

1. $(x + 3)(x - 3)$
2. $(2x + 5)(2x - 5)$
3. $(x - 2)(x - 2)$
4. $(a + 4)(a + 4)$
5. $(x - 4)(x - 3)$

Which of these products are perfect square trinomials?

DEVELOPMENT:

1. Discuss motivation problem. Products 3 and 4 are perfect square trinomials (these problems may be interperted as finding the area of a square, since the two factors are equal). Product 5 is a trinomial but not a perfect square. Products 1 and 2 are binomials which are each the difference of two squares. Ask, "How can you recognize a product of two binomials which will produce a binomial such as this?" Students should observe that the factors are a sum and a difference of the same terms. Why is there no "middle term" in this case? Two of the partial products are opposites and add up to zero.
2. Refer again to the motivation examples. Tell the class that you will now demonstrate an efficient, easy to remember method for writing the product of two binomials called F-O-I-L (First-Outer-Inner-Last).

Consider: $(x + 3)(x - 3)$

F $x \cdot x = x^2$ Multiply the <u>1st</u> terms in each binomial.

O $x(3) = -3x$ Multiply the <u>outer</u> terms.

I $3 \cdot x = 3x$ Multiply the <u>inner</u> terms.

L $(3)(-3) = -9$ Multiply the <u>last</u> terms.

$$(x + 3)(x - 3) = x^2 - 3x + 3x - 9 = x^2 - 9$$

3. Students should apply this method to the other examples:

$$(2x + 5)(2x - 5) = 4x^2 - 10x + 10x - 25 = 4x^2 - 25$$

$$(x - 2)(x - 2) = x^2 - 2x - 2x + 4 = x^2 - 4x + 4$$

$$(a + 4)(a + 4) = a^2 + 4a + 4a + 16 = a^2 + 8a + 16$$

$$(x - 4)(x - 3) = x^2 - 3x - 4x + 12 = x^2 - 7x + 12$$

At this stage the students should write the four partial products as an intermediate step. With practice, they will be able to perform the addition of the middle terms mentally and write only the final result.

4. Verify the correctness of the above results by multiplying the binomials either horizontally or vertically.

For example:

$x + 3$	$2x + 5$	$x - 2$	$a + 4$	$x - 4$
$\underline{x - 3}$	$\underline{2x - 5}$	$\underline{x - 2}$	$\underline{a + 4}$	$\underline{x - 3}$
$-3x - 9$	$-10x - 25$	$-2x + 4$	$4a + 16$	$-3x + 12$
$x^2 + 3x$	$4x^2 + 10x$	$x^2 - 2x$	$a^2 + 4a$	$x^2 - 4x$
$x^2 - 9$	$4x^2 - 25$	$x^2 - 4x + 4$	$a^2 + 8a + 16$	$x^2 - 7x + 12$

5. Consider the algebraic expression: $(m + 2n)(m - n)$. Using the FOIL method:

$$(m + 2n)(m - n) = m^2 - mn + 2mn - 2n^2 = m^2 + mn - 2n^2$$

6. Do Application 1.

7. Students should use multiplication to solve verbal problems. They may use horizontal, vertical or FOIL method.

   a) Find the area of a rectangle with a length of $2x + 7$ and a width of $x - 3$.

   Area = (length)(width)

Consider FOIL:

$$(2x + 7)(x - 3) = 2x^2 - 6x + 7x - 21 = 2x^2 + x - 21$$

   b) An automobile traveled at $2x + 2$ miles per hour for $x + 1$ hours. How far did the auto travel?

   Distance = (rate)(time)

   $= (2x + 2)(x + 1)$

Use the FOIL method:

$$(2x + 2)(x + 1) = 2x^2 + 2x + 2x + 2 = 2x^2 + 4x + 2$$

8. Do Application 2.

APPLICATIONS:

1. Compute using the FOIL method.

   $(x + 2)(x + 3)$ $(x + 2)(x - 5)$

   $(3x - 1)(x + 2)$ $(s + 2t)(s + 2t)$

   $(c + 2d)(c - 2d)$

2. a) Find the area of a rectangle whose length is $3x + 5$ and width is $2x + 4$.

   b) Find the area of a square whose side is $2x - 3$.

SUMMARY:

1. What do the letters in FOIL represent?

2. Explain why the product of two binomials will sometimes have only two terms.

3. Using the FOIL method, state each product:

   $(a + b)(c + d)$ $(a + b)(a - b)$ $(a + b)(a + b)$

HOMEWORK:

1. In Lesson #33 homework examples were solved by horizontal and vertical method. Multiply again using FOIL method.

   $(x + 2)(x - 5)$ $\quad$ $(30 + 2)(4a - 1)$

   $(x - 3)(x + 3)$ $\quad$ $(7a + 2)^2$

   $(3x - 1)(x + 2)$ $\quad$ $(7a + 2)^2$

   $(x - 2)(x + 6)$

2. The length of a rectangle is represented by $x + 3$ and the width by $x + 8$. Express the area of the rectangle as a trinomial in terms of x.

3. Find the area of a square whose side is $2x - 7$.

4. Simplify: $(x - 3)(x - 2) + (x - 2)(x + 5)$

5. Find two numbers whose sum is 10 and whose product is 21.

6. Use the FOIL method to compute the following products and check by giving each variable a numerical value.

   a) $(2a + 3b)(2a - 3b)$ $\quad$ d) $(3x - 2y)(3x - 2y)$

   b) $(4c - d)(4c + d)$ $\quad$ e) $(4u + 3v)^2$

   c) $(7g + 3h)(3g + 7h)$ $\quad$ f) $(2.5 - x)(7.5 + x)$

7. Solve to the nearest integer.

   a) $x^2 = 200$ $\quad$ e) $u^2 - 50 = 0$

   b) $y^2 = 800$ $\quad$ f) $v^2 - 5 = 0$

   c) $r^2 = 300$ $\quad$ g) $w^2 - 12.4 = 0$

   d) $s^2 = 1200$ $\quad$ h) $z^2 - 30.4 = 0$

8. Find two binomials whose computed product is:

   a) $x^2 - 9$ $\quad$ e) $a^2 - b^2$

   b) $y^2 - 1$ $\quad$ f) $4a^2 - 9b^2$

   c) $4x^2 - 1$ $\quad$ g) $a^2 + 2a + 1$

   d) $4r^2 - 9$ $\quad$ h) $a^2 + 2ab + b^2$

Lesson 35 - Common Factoring

AIM: What is meant by factoring?

PERFORMANCE OBJECTIVES: The student will be able to...

1. Define factoring.
2. Use the word factoring correctly in describing expressions.
3. Factor algebraic expressions using greatest common factor.
4. Find the prime factors of integers.

VOCABULARY: product, factors, factoring, distributing

MOTIVATION:

A rectangle has an area of 24 cm^2. What are some possible dimensions?

DEVELOPMENT:

1. Elicit pairs of numbers which could serve as length and width: 24,1 8,3 12,2 6,4
   If students do not suggest noninteger dimensions such as 5,4.8 or 10,2.4, elicit this idea by asking, "Could the rectangle have a length equal to 10cm? Could you have a square with this area?"
   Ask, "How many different rectangles could be formed having an area equal to 24cm^2?" (infinitely many)
   Ask, "How many different rectangles with integer dimensions have this area?" (only the first four listed above)
   The integers 1,2,3,4,6,8,12,24 are called the <u>factors</u> of 24. The process of finding such numbers (or, as we shall see later, variables) is called factoring.
2. What are the factors of 48? List all pairs of positive integers whose product is 48. Examine one set of

factors, i.e., 48 = 12 x 4. Can 12 and 4 be factored? Since 12 = 6 x 2 and 4 = 2 x 2 then 48 = 6 x 2 x 2 x 2. Which of these factors can be factored still further? Since 6 = 3 x 2 then 48 = 3 x 2 x 2 x 2 x 2. Note at this stage we cannot factor the numbers on the right into any other positive integers except by using 1. Review concept of prime and composite numbers and then have class factor 48 in a similar manner by starting with 48 = 16 x 3, or 48 = 24 x 2. Note when we have 'completely' factored 48 into primes, these primes are always the same for each number. (Factor trees may be used to illustrate the process of "complete" factorization.)

3. Ask, "When is it useful to know the factors of integers?" Elicit: Factors are often used when working with fractions. For example, a fraction such as $\frac{6}{8}$ may be reduced as follows:

$$\frac{6}{8} = \frac{2x3}{2x4} = \frac{2}{2} \times \frac{3}{4} = 1 \times \frac{3}{4} = \frac{3}{4}$$

Challenge students to reduce $\frac{48}{60}$ to lowest terms. The class should observe that the most efficient approach is to divide by the <u>greatest</u> common factor.

4. Ask, "What is the best way to find the greatest common factor?" Students may suggest listing all factors of each number: 48: 1,2,3,4,6,8,12,16,24,48

60: 1,2,3,4,5,6,10,12,15,20,30,60

A better method is to factor each number <u>completely</u> (into prime factors), then multiply together the prime factors which are common to both.

We have seen that 48 = 2x2x2x2x3. Have class show that 60 = 2x2x3x5. The greatest common factor (GCF) of 48 and 60 is 2x2x3 or <u>12</u>.

5. Demonstrate that the same techniques can be applied to algebraic expressions, i.e., $6x^2y^3 = 3 \cdot 2 \times x \times y \times y \times y$

$9xy^4 = 3 \cdot 3 \times x \times y \times y \times y \times y$

What is the GCF of $6x^2y^3$ and $9xy^4$? ($3xyyy$ or $3xy^3$)

Students should observe that the coefficient of the GCF

is the GCF of the coefficients, and that each variable appears in the GCF with the smallest exponent shown in either term.

Ask: "What is the GCF of 3x and 9y? (3) Can the expression 3x + 9y be factored?" (Yes, as 3(x + 3y).) This may be checked by multiplying, using the fact that multiplication distributes over addition.)

Have the class describe the procedure for factoring polynimials of more than one term: find the GCF of all terms (this is the first factor); divide the given polynomial by the GCF (this is the second factor).

Apply to other algebraic expressions such as $4x - 6y$, $3x^2 - 6x^3 + 12x$.

APPLICATIONS:

1. Compute: 63 x 97 + 63 x 3 mentally. 63 x 97 + 63 x 3 = 63 (97 + 3) = 63 (100) = 6,300 (Note we have used factoring to compute more easily.)
2. The area of a rectangle is 24xy. If the length is 8x, find the width.
3. The volume of a rectangular solid is $20x^3y^5$. If the length is 2xy and the width is $5xy^2$, find the height.
4. Find the GCF of 42 and 56.
5. Factor:
   a) $3x^2y - 6xy^2$ b) $4x^2y + 12x^3y - 2x$
   c) $48x^2 - 56x^2y$
6. Factor $ax + ay + bx + by$

SUMMARY:

1. What is expected when you are asked to factor an integer? (Factoring is restating a given expression as an equivalent product.)
2. How would you check the answers found for application problem #5? (by multiplying)

HOMEWORK:

1. Compute the following products:

   a) $3a(2a + 3b)$

   b) $4c^2(3 - 5c)$

   c) $(8 - 3)(8 + 3)$

   d) $2x(x^2 + x + 1)$

   e) $3ac^2(4bc - 2ab)$

   f) $(5\frac{2}{3} + 4\frac{1}{3})(5\frac{2}{3} - 4\frac{1}{3})$

2. Factor, showing the G.C.F. as one factor:

   a) $3a + 6b$

   b) $3a + 6a^2$

   c) $4 - 12x$

   d) $x^3 + x^2 + x$

   e) $6a^2c(4bc - 2ab)$

   f) $12(r + s) - 7t(r + s)$

3. Compute two ways, one way by factoring:

   a) $13 \times 7 + 13 \times 3$

   b) $3.1416 \times 76 + 3.1416 \times 24$

   c) $7 \times 2\frac{3}{4} + 7 \times 7\frac{1}{4}$

   d) $9.36 \times 43 - 9.36 \times 33$

4. Solve:

   a) $2a + 3a = 30$

   b) $7x - 3x = 12$

   c) $2a + 3a = 30$

   d) $7x - 3x = 12$

Lesson 36 - Factoring the Difference of Two Squares

AIM: How do we factor expressions of the form $x^2 - a^2$?

PERFORMANCE OBJECTIVES: The student will be able to...

1. Multiply mentally $(x + a)(x - a)$.
2. Recognize an algebraic expression which is the difference of two perfect squares.
3. Factor a difference of two perfect squares.
4. Recognize that $x^2 + a^2$ is not factorable (with real coefficients).

VOCABULARY: perfect square, difference

MOTIVATION:

Which (if any) of the following expressions are equivalent to $x^2 - 9$?

a) $x(x - 9)$
b) $3(x - 3)$
c) $(x - 3)(x - 3)$
d) $(x - 3)^2$
e) $(x + 3)(x - 3)$

DEVELOPMENT:

1. Discuss motivation. Note that both terms of $x^2 - 9$ are perfect squares and we are taking the <u>difference</u> of these two squares. Demonstrate that only $(x + 3)(x - 3)$ is equivalent. When we say that $x^2 - 9 = (x + 3)(x - 3)$, we have <u>factored</u> the expression $x^2 - 9$ by writing it as the indicated product of expressions which have lower degree. This type of factoring is not like "factoring out the GCF". Students should observe that the terms in the expression $x^2 - 9$ have <u>no</u> common factor (they are relatively prime).
   Elicit the generalization: Any difference of two perfect squares may be factored by this rule: $A^2 - B^2 = (A + B)(A - B)$.

2. Provide further practice using examples such as:
   $x^2 - 25$, $x^2 - a^2$, $25a^2 - 36b^2$
   $\frac{4}{49}x^2 - 1$, $x^4 - .01y^2$, $16x^{16} - 49$, etc.
   The last exercise requires particular care:
   $(4x^8 - 7)(4x^8 + 7)$ is correct since $(4x^8)^2 = 16x^{16}$.

3. Can a similar method be used to factor $x^2 + 16$? (No, this is a sum.) Show that suggested factors do not work here.

APPLICATIONS:

1. Factor $49x^2 - 121y^2$

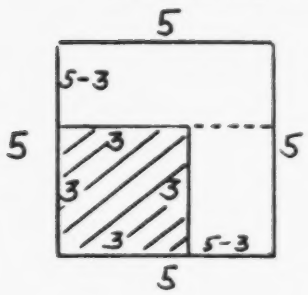
2. Factor $64x^2y^6 - 25n^8$

3. Factor $x^6 - y^{10} = (x^3)^2 - (y^5)^2 = (x^3 + y^5)(x^3 - y^5)$

4. Factor $x^{2a} - y^{2b} = (x^a)^2 - (y^b)^2 = (x^a + y^b)(x^a - y^b)$

5. Multiply 103 x 97 mentally. $(100 + 3)(100 - 3) = 100^2 - 3^2 = 10{,}000 - 9 = 9{,}991$. Note we have used the difference of two squares to simplify.

6. A 3 x 3 square is cut from a 5 x 5 carpet. Show that the remaining carpet contains $(5 - 3)(5 + 3)$ square units.



Area A = $(5 - 3)5 = 10$
Area B = $(5 - 3)3 = 6$

Area A + Area B
$(5 - 3)5 + (5 - 3)3 = (5 - 3)(5 + 3) = (2)(8)$
$= (5^2 - 3^2) = 25 - 9$

The above may also be used to show geometrically that $x^2 - a^2 = (x + a)(x - a)$.

SUMMARY:

1. Factor each expression and explain the difference in strategy used for each:
   $25x^2 - 10y^2$ $\qquad$ $25x^2 - 49y^2$

2. Factor $50x^6 - 18y^2$

HOMEWORK:

1. Compute the product using "multiplication distributing over addition and subtraction":

   a) $8 \times 3\frac{1}{4}$

   b) $6 \times 4\frac{2}{3}$

   c) $12 \times (2\frac{3}{4} - 1\frac{1}{3})$

   d) $30 \times (4\frac{3}{5} + 1\frac{1}{4} - 4\frac{2}{3})$

2. Compute the products:

   a) $(a + b)(a - b)$

   b) $(a + b)(a + b)$

   c) $(3a + 1)(3a - 1)$

   d) $(a - b)(a - b)$

   e) $(1 - 4x)(1 + 4x)$

   f) $(1 + 4x)(1 + 4x)$

3. Factor using the difference of squares and compute:

   a) $8^2 - 7^2$

   b) $51^2 - 49^2$

   c) $1.3^2 - .7^2$

   d) $(5\frac{2}{3})^2 - (4\frac{1}{3})^2$

   e) $111^2 - 89^2$

   f) $10.5^2 - 9.5^2$

4. Factor:

   a) $r^2 - s^2$

   b) $1 - a^2$

   c) $b^2 - 1$

   d) $x^2 - 9$

   e) $16 - y^2$

   f) $4a^2 - 1$

   g) $1 - 4c^2$

   h) $9x^2 - 4y^2$

   i) $(a + b)^2 - (a - b)^2$

5. Factor:

   a) $a^2 - \frac{1}{c^2}$

   b) $\frac{r^2}{s^2} - \frac{s^2}{r^2}$

   c) $x^4 - 1$

   d) $y^4 - 16$

Lesson 37 - Trinomial Factoring

AIM: How do we factor an expression of the form $x^2 + bx + c$?

PERFORMANCE OBJECTIVES: The student will be able to...

1. Factor an expression of the form $ax^2 + bx + c$ where $a = 1$.
2. State the significance of the values and signs of "b" and "c".
3. Check the solution.

VOCABULARY: trinomial, coefficient, term

MOTIVATION:

Multiply:

1. $(x + 5)(x + 3)$
2. $(x + 5)(x - 3)$
3. $(x - 5)(x + 3)$
4. $(x - 5)(x - 3)$

What similarities and differences do you notice in these products?

DEVELOPMENT:

1. Discuss motivation. Elicit that:

   in every case, the product is a trinomial.

   in every case, the absolute value of the constant term is 15, which is the product of 3 and 5.

   When the signs in the factors are the same, the constant term is positive; when the signs in the factors are different, the constant term is negative.

   in every case, the coefficient of the middle term is the sum of the constant terms in the factors.

2. These observations will help us to factor trinomials suc as $x^2 + 5x + 6$. Note (1) there is no common factor and (2) this is not a "difference of two squares". The answer will be in the form $(x + p)(x + q)$. The product pq must equal 6. Ask: "What are the possible ways to factor 6, using integer factors?" (6 x 1 and 2 x 3)

Since the constant term in the trinomial is positive six, we know that p and q must have the same sign. All of these products include a constant term of +6:

$(x + 6)(x + 1)$ $(x + 3)(x + 2)$

$(x - 6)(x - 1)$ $(x - 3)(x - 2)$

Ask: "Which has +5x as the middle term?" (Find each product, using the FOIL method.) Elicit that $(x + 3)(x + 2)$ is the only correct factorization of $x^2 + 5x + 6$.

3. Factor:

   a) $x^2 + 7x + 12$ Note that the possible factors of

   b) $x^2 + 8x + 12$ 12 must be tested to obtain the

   c) $x^2 + 13x + 12$ desired middle term in each case.

4. Factor $x^2 + 5x - 14$. What significance does the minus sign have here in listing possible factors? Elicit that the factors must have opposite signs. Show why $(x - 7)(x + 2)$, $(x + 14)(x - 1)$ and $(x - 14)(x + 1)$ are incorrect. Only $(x + 7)(x - 2)$ produces the exact trinomial desired. Finally, examine examples such as $x^2 - 9x + 20$ (eliciting that the +20 indicates both terms must have the same sign but that -9x tells us that they will be negative) and $x^2 - 2x - 15$ (both signs must be different, as in factoring $x^2 + 5x - 14$ above).

5. A good number of further exercises should be solved and checked, including trinomials where the terms must be rearranged (i.e., $(12 + x^2) - 8x$) and when two variables are involved (i.e., $x^2 + 2xy + y^2$).

APPLICATIONS:

1. Factor $x^2 + 0x - 9$. How does this relate to yesterday's lesson?

2. A rectangle has an area of $x^2 + 6x + 8$. Find its length and width.

3. A rectangular solid has volume of $3x^2 + 18x + 24$. If its height is 3, find its length and width.

SUMMARY:

1. What is the first part of the trinomial you should examine to gain information about its factors? (the constant term)
2. What do all the trinomials considered today have in common? (Note in all cases here the coefficient a = 1. What do you think we will study tomorrow?)

HOMEWORK:

1. Compute the product using the FOIL method and check by assigning a small number to the variable.

a)	$(a + 1)(a + 2)$	e)	$(2c + 1)(2c + 1)$
b)	$(a - 1)(a - 2)$	f)	$(1 - 2c)(1 - 2c)$
c)	$(a - 1)(a + 2)$	g)	$(2c - 3)(2c + 1)$
d)	$(a + 1)(a - 2)$	h)	$(3x - 5)(2x + 4)$

2. Express each trinomial as a product of two binomials (factor):

a)	$a^2 + a - 2$	e)	$e^2 + 7e + 6$
b)	$b^2 + 3b + 2$	f)	$f^2 - 7f + 6$
c)	$c^2 - 3c + 2$	g)	$g^2 + 5g - 6$
d)	$d^2 - d - 2$	h)	$h^2 - 5h - 6$

3. Factor:

a)	$a^2 - 4a$	g)	$y^2 - y - 12$
b)	$b^2 + 9b$	h)	$r^2 - 4 - 3r$
c)	$c^3 + c^2$	i)	$1 - \frac{1}{a^2}$
d)	$c^3 - c$	j)	$1 + \frac{1}{a} - \frac{2}{a^2}$
e)	$x^3 + 2x^2 + x$	k)	$x^2 - x - 30$
f)	$16 - u^4$	l)	$y^2 + 7y - 30$

Lesson 38 - Trinomial Factoring

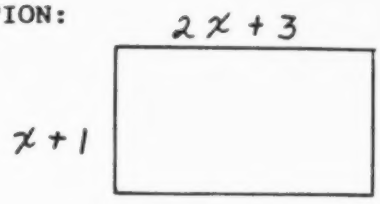
AIM: How do we factor an expression of the form $ax^2 + bx + c$ where $a \neq 1$?

PERFORMANCE OBJECTIVES: The student will be able to...

1. Factor an expression of the form $ax^2 + bx + c$ where $a \neq 1$.
2. State the significance of $a \neq 1$.
3. Check the solution.

VOCABULARY: no new terms

MOTIVATION:



1. Express the area of this rectangle as a trinomial in terms of x.
2. Factor $2x^2 + 5x + 3$.

DEVELOPMENT:

1. Discuss motivation. The students will "see" the solution to problem 2, since this trinomial is the same as the area of the rectangle.
2. Ask: "How does the trinomial we have just factored differ from those factored yesterday?" Recall that in the previous lesson, in factoring trinomials such as $x^2 + 5x + 6 = (x + 2)(x + 3)$, the product of 2 and 3 was the last term of the trinomial (6) while the sum of 2 and 3 was the coefficient of the middle term (5). Is this true for $2x^2 + 5x + 3 = (2x + 3)(x + 1)$? (The constant term <u>is</u> the product of the constant terms in the factors, but the coefficient of the middle term is <u>not</u> their sum.) Elicit that since $a \neq 1$, the coefficients of the "inner" and "outer" products cannot <u>both</u> match the constants in the factors. The coefficient of the middle term of the product will (continue to) be the sum of these coefficients, which will now be <u>different</u> from the sum of the constants in the factors.

3. To factor trinomials such as the above, establish that combinations of all the possible factors of both a and c (in the general form $ax^2 + bx + c$) must be tried to find the combination which produces the correct middle term. The rules for signs are not affected by $a \neq 1$.

4. Consider $3x^2 + 7x - 6$. How would you approach the task of factoring this trinomial? (Note that the terms have no common factor, so we cannot simplify the problem.) Elicit that the solution will be an indicated product of two binomial factors. Since the constant term in the trinomial is <u>negative</u> 6, we know that the constants in the factors will have <u>different</u> signs, and absolute values whose product is 6.
   Observe that 6 may be factored in two ways, as 3 x 2 or 6 x 1. With the signs consideration, this leads to four possibilities:

   $(3)(-2)$ $\qquad$ $(6)(-1)$
   $(-3)(2)$ $\qquad$ $(-6)(1)$

   Also, the coefficients of x in the factors must produce 3 as their product. This <u>combination</u> of conditions leads to eight possibilities, each of which produces a trinomial in which a = 3 and c = -6:

   $(3x + 3)(x - 2)$ $\qquad$ $(3x + 6)(x - 1)$
   $(x + 3)(3x - 2)$ $\qquad$ $(x + 6)(3x - 1)$

   $(3x - 3)(x + 2)$ $\qquad$ $(3x - 6)(x + 1)$
   $(x - 3)(3x + 2)$ $\qquad$ $(x - 6)(3x + 1)$

   Elicit that there are no duplicates in this group. Use the FOIL method to find all eight products. Elicit that each product has a <u>different</u> middle term. Only $(x + 3)(3x - 2)$ produces $3x^2 + 7x - 6$. (This solution may also be written as $(3x - 2)(x + 3)$, since the multiplication is commutative.)

5. Factoring exercises such as the following should then be solved with the class:
   a) $2x^2 - 3x - 2$ $\qquad$ c) $3x^2 + 7x - 6$
   b) $3x^2 - 10x + 3$

6. After a number of such exercises, if time permits, a trinomial where the factors of "a" are not unique such as $4x^2 + 16x + 15$ may be considered. ((2x+5)(2x+3))

APPLICATIONS:

1. The area of a rectangle is $2x^2 + 5x + 3$. Express its length and width in terms of x.
2. The volume of a rectangular solid is $4x^2 + 10x + 6$. If the height is 2, find in terms of x, the length and width. (A discussion of isolating the common factor first - to be studied in tomorrow's lesson - may be included here.)

SUMMARY:

1. Explain why it is more difficult to factor a trinomial of the form $ax^2 + bx + c$ when $a \neq 1$.
2. How would you verify that a trinomial has been factored correctly?

HOMEWORK:

1. Compute using the FOIL method and check by distributing:

a)	$(a + 1)(2a + 1)$	e)	$(2e + 1)(2e + 1)$
b)	$(b + 2)(2b + 1)$	f)	$(2f + 1)(3f + 1)$
c)	$(2c + 1)(c + 2)$	g)	$(3g - 1)(3g + 3)$
d)	$(d + 1)(2d - 1)$	h)	$(5h + 2r)(2h - 5r)$

2. Factor:

a)	$a^2 + a - 2$	g)	$6g^2 + 7g + 1$
b)	$2b^2 + b - 1$	h)	$6g^2 - 5g - 1$
c)	$c^2 - 3c + 2$	i)	$4a^2 - 4a + 1$
d)	$2d^2 - 3d + 1$	j)	$4b^2 + 1 - 4b$
e)	$2e^2 + e - 6$	k)	$4r^2 + 12rs + 9s^2$
f)	$2f^2 - f - 6$	l)	$6x^2 + 11xy - 10y^2$

3. Factor (and evaluate when numerical).

a)	$52^2 - 48^2$	e)	$9s^2 - 1$
b)	$(7\frac{1}{2})^2 - (6\frac{1}{2})^2$	f)	$2u^2 - u - 3$
c)	$14.2857 \times 93 + 14.2857 \times 7$	g)	$3 - v - 2v^2$
d)	$9r^2 - 6r + 1$	h)	$(\frac{u+v}{2})^2 - (\frac{u-v}{2})^2$

Lesson 39 - Complete Factoring

AIM: Can algebraic expressions be factored more than once?

PERFORMANCE OBJECTIVES: The student will be able to...

1. State three types of factoring.
2. State the meaning of "factor completely".
3. Factor expressions completely.

VOCABULARY: completely factored

MOTIVATION:

Factor each expression:

1. $25x^2 - 40x$
2. $25x^2 - 49$
3. $5x^2 - 6x + 1$
4. $x^2 + x + 3$

Did you use the same strategy for each problem?

DEVELOPMENT:

1. Discuss motivation. Elicit that the first two expressions are binomials, each containing the term $25x^2$, yet a different method is used in each:
   the terms in $25x^2 - 40x$ have a common factor, $25x^2 - 40x = 5x(5x - 8)$.
   the second expression is a difference of two perfect squares, $25x^2 - 49 = (5x + 7)(5x + 7)$.
   Elicit that expressions 3 and 4 are trinomials. Each has a positive constant term, indicating that the constants in the factors will have the same sign (both positive or both negative). The first can be factored as $(5x - 1)(x - 1)$. $x^2 + x + 3$ <u>cannot be factored</u>, since $(x + 3)(x + 1)$ and $(x - 3)(x - 1)$ each produce a "middle term" different from $+1x$.
2. Consider $2x^2 + 4x + 2$. Is it possible to factor this trinomial? In this exercise, students will often suggest

$(2x + 2)(x + 1)$ as the answer; others will note the common factor and obtain $2(x^2 + 2x + 1)$. Ask: "Is the problem finished?" Elicit that in each case one factor can be factored; i.e., that the "factoring" process can continue.

$2x^2 + 4x + 2$	$2x^2 + 4x + 2$
$(2x + 2)(x + 1)$	$2(x^2 + 2x + 1)$
$2(x + 1)(x + 1)$	$2(x + 1)(x + 1)$

As with integers, it should be noted here that regardless of the techniques employed, the final "completely factored" form will be the same (except, possibly, for the order of the factors).

Ask: "How do you know when you have factored an expression completely ?" Elicit: When each factor is an expression which cannot itself be factored, such as $x + 1$ or $x^2 + x + 3$. (These may be called "prime polynomials". All monomials are considered "prime" in this regard.) An analogy may be made with "reducing" fractions and "reducing fractions completely".

3. Ask the students to factor $25x^2 + 30x + 5$. Is it easier to factor out the greatest common factor first or to try to write this expression as the product of two binomials? Students should conclude that since it is easier to factor a trinomial with small coefficients, the best approach is to begin by examining the terms in the hope of finding a common factor;

   $25x^2 + 30x + 5 \longrightarrow 5(5x^2 + 6x + 1) \longrightarrow 5(5x + 1)(x + 1)$

   will generally take less time than routes through $(5x + 1)(5x + 5)$ or $(25x + 5)(x + 1)$ to the same goal.

4. Factor completely:

   1) $5x^2 - 45$
   2) $2x^2 - 8x + 6$
   3) $3x^2 - 3$
   4) $x^4 - y^4$

APPLICATIONS:

Factor completely:

1. $2x^2 + 8x + 6$
2. $25x^2 - 100$

3. $4x^2 + 10x + 6$
4. $ax^2 - ay^2$
5. $16x^4 - y^8$
6. $15x^2y - 18x + 3$
7. $18x^2 + 50y^2$

SUMMARY:

1. Given a polynomial to factor, how would you begin?
2. When is the "factoring" process finished?

HOMEWORK:

1. Factor completely:
   a) $2r^2 - 2s^2$
   b) $9u^2 - 9v^2$
   c) $9u^2 - 4v^2$
   d) $u^2 - 1$
   e) $ar^2 - as^2$
   f) $2x^2 + 4x + 2$
   g) $3r^2 - 6r + 3$
   h) $5 + 5s - 30s^2$
   i) $4r^2 - 16 - 12r$
   j) $15 + 5u - 10u^2$
   k) $rs^2 - r$
   l) $2x^4 - x^2 - 1$
2. Using factoring, compute:
   a) $9^2 - 8^2$
   b) $27^2 - 24^2$
   c) $3.14 \times 51^2 - 3.14 \times 49^2$
   d) $(6\frac{3}{4})^2 - (3\frac{1}{4})^2$
   e) $2700^2 - 2300^2$
   f) $30^2 - 2 \times 29 \times 30 + 29^2$
3. If the domain of each variable is the set of integers from -10 to 10 inclusive, solve:
   a) $x^2 = 25$
   b) $4a^2 = 100$
   c) $c^2 + c - 6 = 0$
   d) $x^2 < 9$
   e) $2r^2 + r - 6 = 0$
   f) $2x^2 + x - 12 \leq 0$

Lesson 40 - Quadratic Equations

AIM: What is a quadratic equation?

PERFORMANCE OBJECTIVES: The student will be able to...

1. Define "quadratic equation".
2. State the zero product property.

VOCABULARY: quadratic equation, standard form

MOTIVATION:

Solve for x: $x + 9 = 0$ $9x = 0$

What conclusion can be drawn when the sum of two numbers is zero? What conclusion can be drawn when the product of two numbers is zero?

DEVELOPMENT:

1. Discuss motivation. Elicit: When two numbers add up to zero, they are opposites (additive inverses). Ask the students to name pairs of numbers whose sum is zero. Ask the students to name pairs of numbers whose product is zero. Elicit: When the product of two numbers is zero, at least one of the numbers must be zero. Using logical notation, we may write $(AB = 0) \longrightarrow (A=0 \text{ V } B=0)$. Is the converse true? (yes) Therefore the antecedent and consequent are equivalent statements and we may write a biconditional: $(AB=0) \longleftrightarrow (A=0 \text{ V } B=0)$. A zero product is obtained if and only if at least one factor is zero.
2. Present two more equations for consideration:

   $x^2 - 9 = 0$ $x^2 + 8x - 9 = 0$

   Ask, "How do these equations differ from the first two which were considered?" Elicit: Each of these equations involves x <u>squared</u> as the highest noted power of x. Tell the students that such an equation is called a <u>quadratic equation</u>. Should $x^2 + 8x = x^2 + 16$ receive this title? (No, when this equation is expressed more simply after

combining like terms, it is found to be equivalent to $8x = 16$ or $x = 2$, which do not involve x squared.)

3. Challenge the students to find the solution set for each of the quadratic equations cited above. Students should observe that $x^2 - 9 = 0$ is equivalent to $x^2 = 9$. This equation has two roots, +3 and -3. (Verify that each satisfies the original equaiton.)

   $(x^2 - 9 = 0) \longleftrightarrow (x=+3 \text{ V } x=-3)$

   $(x^2 - 9 = 0) \longleftrightarrow (x \pm 3)$

   At this point the students may not be able to identify either root of $x^2 + 8x - 9 = 0$.

4. Demonstrate how the zero product rule may be applied to find the roots of $x^2 - 9 = 0$ by using the "difference of two squares" rule to factor the left side of the equation:

$$x^2 - 9 = 0$$
$$(x - 3)(x + 3) = 0$$
$$x - 3 = 0 \text{ or } x + 3 = 0$$
$$x = +3 \text{ or } x = -3$$

   Ask: "Can this method be used to find the solution set of $x^2 + 8x - 9 = 0$?" Ask: "How many roots do you think this equation will have?"

   Solve $x^2 + 8x - 9 = 0$ by the factoring method. The solution set is $\{-9,+1\}$ since

$(x^2 + 8x - 9 = 0) \longleftrightarrow ((x+9)(x-1) = 0) \longleftrightarrow (x+9=0 \text{ V } x-1=0) \longleftrightarrow (x=-9 \text{ V } x=+1)$

5. Present $x(x + 1) = 56$. Ask: "Is this a quadratic equation?" Students should observe that the equation is equivalent to $x^2 + x = 56$ and is a quadratic equation. Elicit: In order to make use of the above property of zero, we must have a product equal to zero. Therefore, we must transform the equation to the equivalent $x^2 + x - 56 = 0$ and then factor the left member. The above form is referred to as the standard form of a quadratic; e.g., $ax^2 + bx + c = 0$. The class should then factor to obtain $(x + 8)(x - 7) = 0$, express this idea

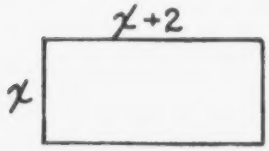
equivalently as $(x + 8 = 0)$ or $(x - 7 = 0)$, and solve these to obtain the roots: $x = -8$ or $x = +7$. Check each root in the original equation.

APPLICATIONS:

1. Transform each quadratic equation into standard form:
   a) $x^2 = 8x + 5$
   b) $x^2 = 49$
   c) $x^2 - 8x = 3x^2 + 5x - 2$
   d) $x(x + 3) = 2x + 4$
   e) $5x = 2x^2 - 1$
   f) $-6x = 5 - x^2$

2. Which of these are quadratic equations? Explain.
   a) $x^2 = x(x + 5)$
   b) $x^2 - 5 = 8x$
   c) $x^2 - 5x = x(x - 5)$
   d) $2x(x + 5) = x(x - 1)$

3. Solve for x:
   a) $x^2 + 6x + 9 = 0$
   b) $x^2 - 25 = 0$
   c) $2x^2 + 10x + 12 = 0$
   d) $x^2 = 4x + 21$

4. 

The area of the rectangle shown is 24; write a quadratic equation which describes this situation.

5. The square of a number is 3 more than half the number. Write a quadratic equation which describes this situation.

SUMMARY:

1. What is a quadratic equation?
2. What procedure is used to find the solutions of a quadratic equation?
3. Why should you transform quadratic equations into standard form?
4. Will every quadratic equation have two roots? (No: Consider $x^2 - 8x + 16 = 0$.)

HOMEWORK:

1. Which of the following are quadratic equations?

   a) $x^2 + 1 = x^3$
   b) $r^2 - 1 = 0$
   c) $s^2 + s = 6$
   d) $y - 9 = 0$
   e) $1 + x = x^2$
   f) $u^2 + \frac{1}{u} = 5$

2. Compute: $9 \times 8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1 \times 0$.

3. Solve:

   a) $(x - 1)(x - 2) = 0$
   b) $(y - 7)(y - 11) = 0$
   c) $(u + 1)(u - 3) = 0$
   d) $(2r - 1)(3r - 1) = 0$
   e) $(s - 1)(s - 2)(s - 3) = 0$
   f) $x(x + 2) = 0$
   g) $p(p + 1) = 2$
   h) $5(2c - 3)(3c + 2) = 0$

4. Factor; then solve:

   a) $x^2 - 3x + 2 = 0$
   b) $p^2 - 16 = 0$
   c) $2u^2 - 18 = 0$
   d) $v^2 - 2v + 1 = 0$
   e) $2r^2 + 3r + 1 = 0$
   f) $5s^2 + 10s = 0$
   g) $10t = 2t^2$
   h) $x(x + 2) + 1 = 0$

Lesson 41 - Quadratic Equations

AIM: How do we solve a quadratic equation?

PERFORMANCE OBJECTIVES: The student will be able to...

1. State the recommended procedure for solving quadratic equations.
2. Solve quadratic equations.
3. Check the solution.

VOCABULARY: no new terms

MOTIVATION:

Which equation is easier to solve? Explain.

1. $x(x + 2) = 0$ 2. $x(x + 2) = 24$

DEVELOPMENT:

1. Discuss motivation. Elicit that the first equation is easier to solve because it already shows a product equal to zero. Students should recall from yesterday's lesson that this implies that one factor must be zero, $x = 0$ or $x + 2 = 0$ (i.e., $x = -2$).
2. Recall: These are quadratic equations. Elicit the procedure (learned yesterday) for solving the second equation. Summarize in writing as each step is applied to this equation.

Procedure	
$x(x + 2) = 24$	1. Place the equation into standard form. $(ax^2 + bx + c = 0)$
$x^2 + 2x = 24$	
$x^2 + 2x - 24 = 0$	
$(x + 6)(x - 4) = 0$	2. Factor the trinomial.
$x + 6 = 0$ or $x - 4 = 0$	3. Set each factor equal to 0.
$x = -6$ or $x = +4$	4. Solve each equation.

Solution set: $\{-6, +4\}$

Check	Check	
$x(x + 2) = 0$	$x(x + 2) = 24$	5. Check each root in the original equation.
$-6(-6 + 2) \stackrel{?}{=} 24$	$4(4 + 2) \stackrel{?}{=} 24$	
$-6(-4) \stackrel{?}{=} 24$	$4(6) \stackrel{?}{=} 24$	
$+24 = 24$	$24 = 24$	

APPLICATIONS:

1. $x^2 - 7x + 10 = 0$
2. $x^2 - 6x + 9 = 0$
3. $x^2 = - x + 6$
4. $x^2 = 16$
5. $x^2 = \frac{x}{2} + 3$ (from previous lesson, "Application" #5)
6. $x^2 - x = 0$

SUMMARY:

1. What are the steps used to solve a quadratic equation?
2. Find the error in the following:

$x^2 - 5x + 6 = 0$

$(x - 3)(x - 2) = 0$

$x - 3 = 0$	$x = 3$
$x - 2 = 0$	$x = 2$

Check

$x^2 - 5x + 6 = 0$

$3^2 - 5(2) + 6 \stackrel{?}{=} 0?$

$9 - 10 + 6 \stackrel{?}{=} 0$

$-1 + 6 \stackrel{?}{=} 0$

$+5 \stackrel{?}{=} 0$ (false)

HOMEWORK:

1. What conclusion may be drawn from each of the following (a solution is not called for):

a) $xy = 0$	d) $(2r + r - 3)(5r - r - 4) = 0$
b) $xy > 0$	e) $x^2 < 0$
c) $xy < 0$	f) $x^2 + y^2 = 0$

2. Solve and check:
   a) $(x^2 - 9)(2x - 1) = 0$
   b) $(y^2 + y - 6)(3y + 1) = 0$
   c) $(\frac{4}{u} - 2)(u + 1) = 0$

d) $r^2 = r + 6$

e) $s(s + 2) = 15$

f) $2w^2 - w + 3 = 0$

g) $7p^2 + 2p - 5 = 0$

h) $5 = 3p + 8p^2$

i) $2x^2 - x = 15$

j) $v^2 - v = 42$

k) $w^2 + 2w = 35$

l) $6x^2 - x - 12 = 0$

3. Find two consecutive odd integers whose product is 63.

4. A certain rectangle is three feet longer than it is wide. If its area is 54 sq. ft., what is its perimeter?

5. A certain number increased by five times its reciprocal is 6. What could this number be?

Lesson 42 - Verbal Problems Solved by Quadratics

AIM: How do we solve an integer problem leading to a quadratic equation?

PERFORMANCE OBJECTIVES: The student will be able to...

Solve a consecutive integer problem leading to a quadratic equation.

VOCABULARY: Consecutive integers, sequence, consecutive even, consecutive odd

MOTIVATION:

Find two consecutive integers whose product is 56.

DEVELOPMENT:

1. Discuss motivation. Students may observe that the pair 7,8 satisfies the conditions specified. Is this the only pair of consecutive integers that works? Review the use of algebraic expressions to represent a sequence of consecutive integers. How could we express this condition algebraically? Elicit the equation $x(x + 1) = 56$. This equation has two solutions: +7 and -8, leading to <u>two</u> pairs of consecutive integers in the solution set for this problem: 7 and 8, and also -8 and -7.

2. Challenge the students to find three consecutive integers such that the square of the smallest is equal to the sum of six times the middle integer plus the largest integer. Elicit that here $x^2 = 6(x + 1) + (x + 2)$ and that this is a quadratic equation.
   After obtaining the standard form of the above, $x^2 - 7x - 8 = 0$, solve the equation, and ask the class to interpret the two solutions $x = 8$ or $x = -1$. The first yields the result 8, 9, 10, while the second yields -1, 0, 1. In this problem, both sets of numbers are acceptable as solutions. The class should be informed that many problems ask only for the positive integers and

care must be taken in reading the problem so that improper solutions may be "discarded".

3. Find three consecutive odd integers such that the sum of the first and the square of the third is 128.
   A format including legend, equation, solution, answer sentence, and check in the statement of the problem should be demonstrated and encouraged. Elicit that x, x + 2, x + 4 may be used to represent the three consecutive odd integers.

$$x + (x + 4)^2 = 128$$
$$x + x^2 + 8x + 16 = 128$$
$$x^2 + 9x - 112 = 0 \quad \text{(standard form)}$$
$$(x + 16)(x - 7) = 0$$
$$x = -16 \text{ or } x = 7$$

   Although both of these values will make the equation true, -16 does not satisfy the specified conditions because it is not odd. The only solution triple is 7,9,11.

Check: $7 + 11^2 \stackrel{?}{=} 128$
$7 + 121 = 128$

APPLICATIONS:

1. Find three consecutive even integers such that the product of the first and the second is 6 less than nine times the largest.
2. The length and width of a rectangle may be represented by two consecutive odd integers. If the area is 35 sq. units, find the length and width algebraically.
3. A rectangular solid has length, width and height which can be represented by consecutive even integers. If the smallest dimension is increased by 2 and the largest decreased by 2, what type of solid will be formed?

SUMMARY:

1. Can the methods used today be used to find two consecutive integers whose product is 51? Explain. (No,

one of every two consecutive integers must be even, so the product will be even. It is impossible to find two consecutive integers whose product is an odd number such as 51.)

2. Instruct the class to write three consecutive integers (or consecutive even or odd integers) in their notebooks. Then have each student construct a problem involving a quadratic equation to find the chosen numbers if some relationship between the numbers and nature of the integers is specified. The problems should be collected and used for additional practice.

HOMEWORK:

1. The area of a rectangle is 48. If its dimensions are represented by 2a and 3a - 1, find numerical values for its dimensions.
2. The sum of the squares of two consecutive integers is 113. Find them.
3. The ratio of two numbers is 2:3. If their product is 54, what might these numbers be?
4. Write a quadratic equation in standard form if its roots are:

a)	2 and 3	e)	5 and 5
b)	-2 and -3	f)	5 and -5
c)	2 and -3	g)	2 and $\frac{1}{2}$
d)	-2 and 3	h)	$\frac{1}{2}$ and $\frac{1}{3}$

Lesson 43 - Verbal Problems Solved by Quadratics

AIM: How do we solve a number problem leading to a quadratic equation?

PERFORMANCE OBJECTIVES: The student will be able to...

Solve a number problem leading to a quadratic equation.

VOCABULARY: no new terms

MOTIVATION:

The teacher may announce, "I am thinking of an integer whose square is 15 more than twice the number." The class may be separated into two groups which will try to find the number by 1) guessing and 2) algebraically.

DEVELOPMENT:

1. Discuss motivation. The "guessers" may have found the solution "5" more quickly, but are unlikely to have uncovered the second root, "-3". Students should observe that the algebraic method has the advantage of exposing every possible solution. The necessity of checking each solution in the statement of the problem should be emphasized.
2. Consider as many of the following problems as time permits:

APPLICATIONS:

1. The square of a number is 8 greater than twice the number. Find the number.
2. The sum of the square of a number and half the number is 39. Find the number.
3. Twice the square of a number is equal to six times the number. Find the number.
4. Two numbers are in the ratio of 3:5. If their product is 60, find the numbers.

5. The sum of two numbers is 8. The sum of their squares is 40. Find the numbers.

6. Al is two years older than Beth. If the product of their ages is 63, find their ages.

7. "When is the square of a number larger than the number?" Have students give examples. Solve $x^2 = x$ and discuss $x^2 < x$. ($x^2 < x$ when $0 < x < 1$.)

SUMMARY:

Ask the students to each pick an integer and make a quadratic number problem. Select one and have the class solve the problem.

HOMEWORK:

1. If 4 times the square of the middle of three consecutive integers equals the sum of the squares of the other two integers, what might they be?

2. If a root of the equation $x^2 + x - K = 0$ is 3, what must the value of K be? What must the other root be?

3. If -2 is a root of $3x^2 + Kx - 6 = 0$, what must the other root be?

4. If the sum of the squares of two consecutive even integers is 100, what might they be?

5. The tens' digit of a number is the square of its units' digit. If the number is a two-digit number and the sum of its digits is 12, what is this two digit number?

6. The sum of two numbers is 2 and their product is -35. What is a quadratic equation that has for its roots these two numbers?

Lesson 44 - Verbal Problems Solved by Quadratics

AIM: How do we solve an area problem leading to a quadratic equation?

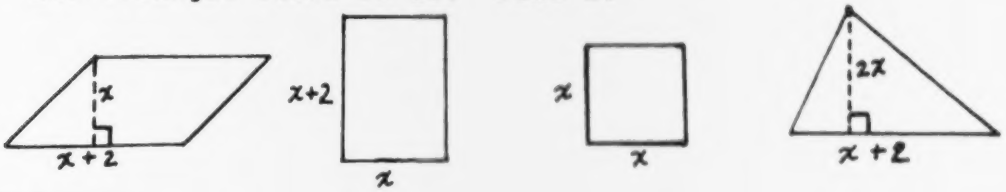
PERFORMANCE OBJECTIVES: The student will be able to...

Solve an area problem leading to a quadratic equation using parallelograms, rectangles, squares and triangles.

VOCABULARY: parallelogram, triangular prism, altitude, base

MOTIVATION:

The sum of the area of the parallelogram, rectangle, square and triangle shown is 28. Find x.



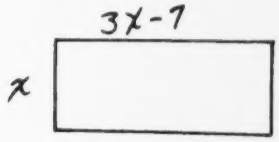
DEVELOPMENT:

1. Review area formulas, have class obtain area of each figure in terms of x. Sum of areas:

$$x(x + 2) + x(x + 2) + x^2 + \frac{1}{2}(2x)(x + 2) = 28$$
$$2x^2 + 3x - 14 = 0$$
$$(2x + 7)(x - 2) = 0$$

Discuss why x cannot be $\frac{-3}{2}$. Check x = 2.
Which figures here have equal areas? (parallelogram, rectangle, triangle)

2. In a rectangle whose area is 40 sq. cm, the length is 7 cm less than 3 times the width. Find the dimensions of the rectangle.
Elicit: A diagram would facilitate visualizing this situation.



$x(3x - 7) = 40$

$3x^2 - 7x - 40 = 0$

$(3x + 8)(x - 5) = 0$

$3x + 8 = 0 \qquad x - 5 = 0$

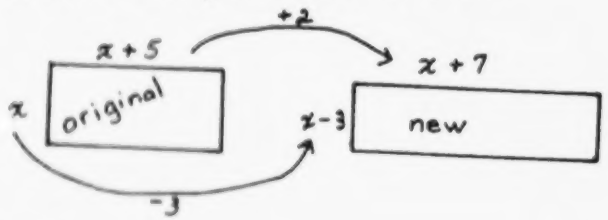
$x = -\frac{8}{3} \qquad x = 5$

(discard)

The width is 5 cm and the length is 8 cm.

3. We have just seen how quadratic equations may result from the areas of different geometric figures. Elicit the procedure for solving a problem of this type. Begin with a diagram. We must then represent the unknown lengths in terms of a variable, i.e., set up a legend and label the diagram in terms of that variable. We then translate the problem into an equation and solve and check. Negative solutions cannot serve as lengths and must be discarded.

4. Consider: The length of a rectangle is 5 more than its width. If the length is increased by 2 and the width is decreased by 3, a new rectangle is formed whose area is 56 square inches. Find the area and perimeter of the original rectangle.



$(x - 3)(x + 7) = 56$

$x^2 + 4x - 21 = 56$

$x^2 + 4x - 77 = 0$

$(x + 11)(x - 7) = 0$

$x = -11$ (reject) or $x = +7$

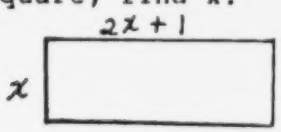
The original rectangle has dimensions 7" and 12", area 84 square inches, perimeter 38". (The second rectangle has dimensions 4" and 14".)

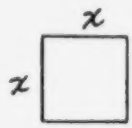
APPLICATIONS:

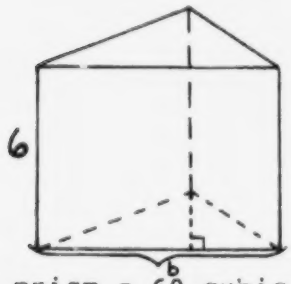
1. The base of a parallelogram is 5 more than the altitude to that base. If the area is 24, find the altitude and base.

2. The altitude to the base of a triangle is 2 less than the base. Find each if the area of the triangle is $7\frac{1}{2}$ square units.

3. If the area of the rectangle is 4 more than that of the square, find x.





4. 

The volume of a right triangular prism is the area of the base (a triangle) times the height of the prism. Given the following information, find the altitude h and base b of the triangle.

Vol. prism = 60 cubic units
height prism = 6 units
b (base of triangle) is 1 more than h (altitude of triangle)

SUMMARY:

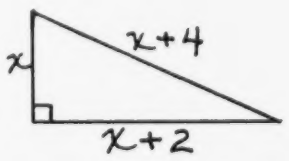
Compare the following problems involving quadratics.

1. The product of two consecutive even integers is 24. Find the integer.
2. One number is 2 more than another. Their product is 24. Find each number.
3. The base of a rectangle is 2 more than the height. The area is 24. Find the base, height.

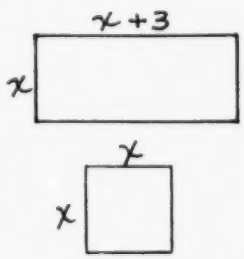
What do you notice? Solve all "three" problems.

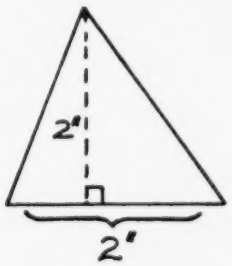
HOMEWORK:

1. Find the numerical area of the right triangle shown. (Note: First find the value of x.)



2. Suppose the sum of the areas of the rectangle and square is equal to the area of the triangle. Find the dimensions and areas of the quadrilaterals.





3. Find a, b, c for the quadratic equation $ax^2 + bx + c = 0$ when the roots are:

   a) 7 and -7
   b) 10 and -10
   c) $\frac{1}{2}$ and $-\frac{1}{2}$
   d) r and -r
   e) What seems to be the relationship among a, b, c when the sum of the roots is 0?
   f) Is the converse of your answer to e) true?

4. Find a, b, c for the quadratic equation $ax^2 + bx + c = 0$ when the roots are:

   a) 2 and $\frac{1}{2}$
   b) 3 and $\frac{1}{3}$
   c) $\frac{2}{3}$ and $\frac{3}{2}$
   d) $-\frac{2}{3}$ and $-\frac{3}{2}$
   e) What seems to be the relationship among a, b, c when the product of the roots is 1?
   f) Is the converse of your answer to e) true?

5. If the perimeter of a rectangle is 48 ft., which dimensions seem to give the greatest area? You may want to experiment with some numbers as 2 ft. by 10 ft., 3 ft. by 9 ft., etc.

6. Try to generalize question 5 and provide a proof.

Lesson 45 - Circumference of a Circle

AIM: How do we find the circumference of a circle?

PERFORMANCE OBJECTIVES: The student will be able to...

1. State what is meant by the circumference of a circle.
2. State the formulas for circumference.
3. Find the circumference of a circle, given the radius or diameter.
4. Find the radius given the circumference.

VOCABULARY: circumference

MOTIVATION:

Using containers with rectangular (cardboard box) and circular (waste basket) tops and a tape measure (or string and ruler, if tape measures are not available), the teacher should demonstrate how to measure the length of the edge of the opening. Elicit that for the rectangle this measurement is called the perimeter. Announce that for the circle the measurement is called the circumference.
The teacher should then demonstrate how to use the tape (or string) to measure the diameter of the circle (longest chord). The class should be divided into groups, and each group given one or more circular objects (hula hoop, coffee cup, oatmeal box, etc.): measure the circumference and diameter of the circle.

DEVELOPMENT:

1. Tabulate the results. Students should observe that in every case the circumference is slightly more than three times the diameter. Divide to find the $\frac{C}{D}$ ratio in each case. The teacher should explain that the differences are due to approximations caused by the rough measurement methods used. If perfectly precise measurements were

made, the $\frac{C}{D}$ ratio would be exactly the same in every case. This constant is an irrational number (Review: a nonterminating, nonrepeating decimal), approximately equal to 3.1415926536. Computers have been used to calculate thousands of decimal places of this number. The name commonly used to refer to the number is "pi", the name of the Greek letter π. This letter was used for the sound we represent by "P", and was chosen to represent this number to emphasize the connection with the "perimeter" of any circle. A rational number close to the value of π is $3\frac{1}{7}$ ($\frac{22}{7}$), which has the decimal value $3.142857\overline{142857}$. $\frac{22}{7}$ and 3.14 are commonly used as <u>approximations</u> of the value of π.

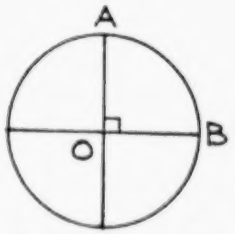
2. From $\frac{C}{d} = \pi$, obtain $C = \pi d$. Develop: Since $d = 2r$, $C = 2\pi r$. This formula may be used to find the length of the circumference when the radius is known.

3. Exercises such as the following should be presented. (Answers may be approximated or left in terms of π.)
   1) If the radius of a circle is 4, find the circumference.
   2) If $d = 10$, find C.
   3) If $C = 36\pi$, find r and d.
   4) If r is tripled, how does C change?

      $C_1 = 2\pi r$

      $C_2 = 2\pi(3r) = 6\pi r$

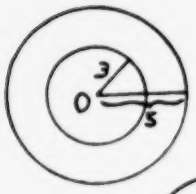
6. Find the length of (minor) arc AB if OB = 6 where O is the center of the circle.



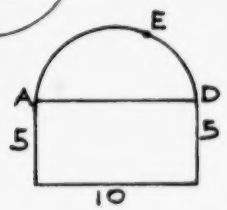
7. A wheel is rotated three times along a road without slipping. If the diameter equals 2 ft., find the distance the center of the wheel travels. How many times must it rotate to travel 100 ft.?

APPLICATIONS:

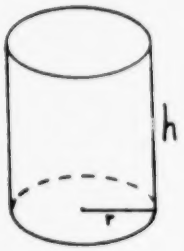
1. How much longer is the circumference of the outer circle than the inner? (Both are concentric with center O.)



2. Find the perimeter of the figure shown. ($\overset{\frown}{AED}$ is a semicircle.)



3. (Discuss the concept of surface area first.) A right circular cylinder has a lateral surface area S equal to the circumference of its base (a circle) times the height. What is the formula for S? If r = 2 and h = 3, find S.



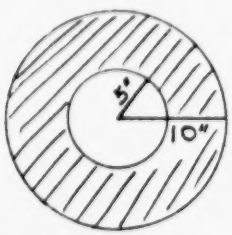
SUMMARY:

1. What does circumference mean?
2. What is the relationship between the diameter of a circle and its circumference?
3. What is the relationship between the radius of a circle and its circumference?
4. What is π?

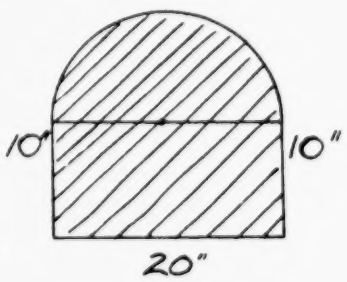
HOMEWORK:

1. For each of the following, compute the perimeter of the shaded region in terms of π. Using 3.14 for π, estimate to the nearest whole number your answer.

a)

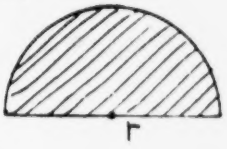


b)

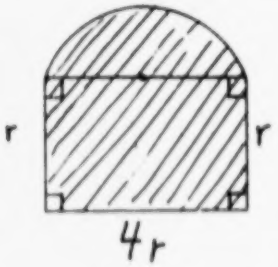


2. In terms of π and r, express the perimeter of the shaded region.

a)

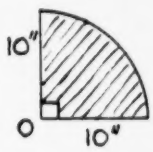


b)

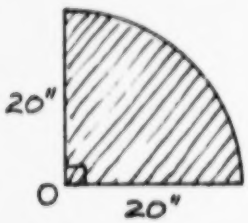


3. Compute the perimeter of the shaded region to the nearest whole number using 3.14 for π.

a)



b)



How do your answers in a) and b) compare?

4. Do example 3 using a 60° angle at 0 instead of a 90° angle.

5. The circumference of a circle is 1000m. Find to the nearest meter its radius. (Use 3.14 for π.)

*6. The perimeter of a semicircle is 1000m. Find to the nearest meter its radius. (Use 3.14 for π.)

Lesson 46 - Area of a Circle

AIM: How do we find the area of a circle?

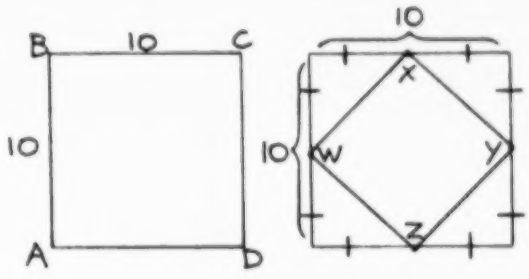
PERFORMANCE OBJECTIVES: The student will be able to...

1. State the formula for the area of a circle.
2. Describe the nature of π.
3. Find the radius of a circle given its area.

VOCABULARY: irrational, right circular cylinder, concentric circles (optional), sphere, surface area, semicircle.

MOTIVATION:

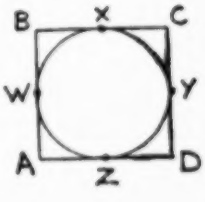
Find the area of square ABCD.
Find the area of square WXYZ.



DEVELOPMENT:

1. Discuss motivation. Square ABCD has area = 100. Square WXYZ has half this area (area = 50). This may be calculated directly (by the Pythagorean theorem, WX = $5\sqrt{2}$; $(5\sqrt{2})^2 = 25 \cdot 2 = 50$) or inferred by drawing additional lines in the figure.

2. Today we will try to find a formula for the <u>area</u> of a circle. Consider a circle with diameter 10. It would just fit inside square ABCD, touching the sides at points W, X, Y, Z.
The area of the circle is less than the area of square ABCD but greater than the area of square WXYZ.
Generalizing, if the circle has radius = r, it fits inside a square whose side is 2r (Area = $4r^2$) and contains a square whose diagonal is 2r (Area = $2r^2$).



A resonable rough approximation appears to be $3r^2$.

3. Consider the area of the circle to be cut up like a pizza:

If the slices were rearranged like this:

a figure resembling a parallelogram would be formed (the more slices, the more like a parallelogram it would be). What is the "height" of this "quadrilateral"? (r) What is the "base"? ($\frac{1}{2}$ circumference or $\frac{1}{2}(2\pi r)$ or πr)
By the formula for the area of a parallelogram, the area of this reassembled pizza would be base x height = $(r)(\pi r)$ or πr^2.

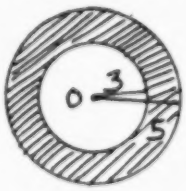
4. Challenge the students to find a formula for the area of a circle in terms of its diameter. Since $d = 2r$, $r = \frac{d}{2}$ and $A = \pi r^2$ becomes $A = \frac{\pi d^2}{4}$.

5. Exercises such as the following may be considered:
   1) Find A if diameter = 6 (answers may be approximated or left in terms of π).
   2) If $A = 49\pi$, find r. Find d. Find C.
   3) Find the area of a semicircle of radius 8.

6. (Optional) The formula for the surface area of a sphere is $S = 4\pi r^2$. Find the surface area if the radius of the sphere is 5.

APPLICATIONS:

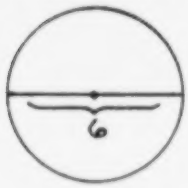
1. Find the shaded area

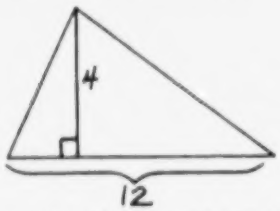


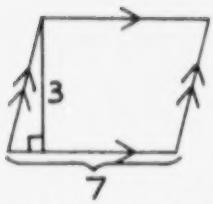
(Both circles have center O. They are called <u>concentric</u> circles.)

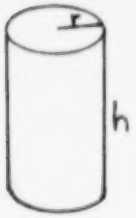
"Is $\pi 5^2 - \pi 3^2 = \pi 2^2$?" (NO)

2. Which has the greatest area?







3. 

A right circular cylinder has a volume equal to the area of its base (a circle) times the height i.e., $V = \pi r^2 h$. If $r=2$ and $h=3$, find V.

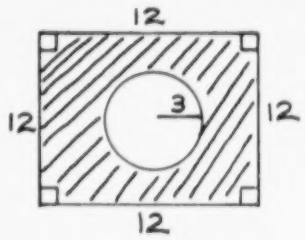
SUMMARY:

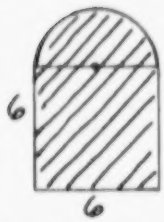
1. State the formulas for the area and circumference of a circle.
2. What kind of number is π?
3. Describe how you would find the area of a sector of a circle whose central angle is 45°.

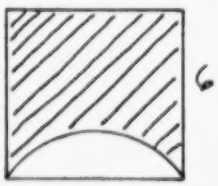
HOMEWORK:

1. In terms of π find the area and perimeter of the shaded regions. Then approximate both answers to the nearest whole number, using 3.14 for π.

a)

b) 

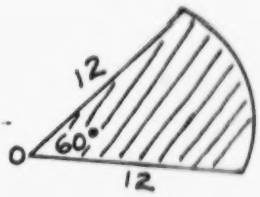
c) 

d) 

2. In terms of π find the area and perimeter of the shaded region.

a)

b)



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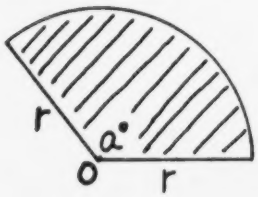
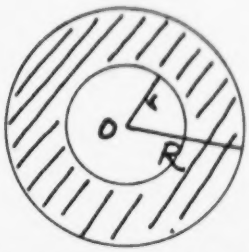


Express area in terms of a, r and π.

d)



Express area in terms of r, R and π.

Lesson 47 - Changes in Area Resulting from Changes in Linear Dimensions

AIM: How does changing a linear dimension of a figure affect its areas?

PERFORMANCE OBJECTIVES: The student will be able to...

Solve problems related to changes in the area of triangles, quadrilaterals and circles as a result of changes in their linear parts.

VOCABULARY: no new terms

MOTIVATION:

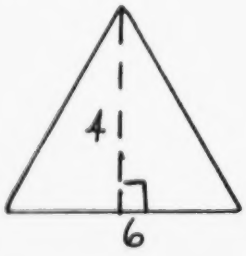
If a 6" circular pizza (6" is the diameter) costs $1.50, how much should a 12" pizza cost?

DEVELOPMENT:

1. Students should recall the formula for the area of a circle. After computing the areas 9π and 36π, they should see that the larger pie has <u>four</u> times the amount of pizza although the diameter was simply <u>doubled</u>. (Note: Substituting an approximate value for π does <u>not</u> improve understanding here. The <u>exact</u> values 9π and 36π show that the ratio of areas is exactly four much more quickly and clearly.) The price for the large pie should be 4 x $1.50 or $6.00.

2. The materials needed to construct a medallion in the shape of the triangle shown is $5.

What would be the cost of materials for a similar triangle such as the following? Students should recall $A = \frac{1}{2}bh$ to obtain areas of 3 and 12 respectively. The area has been multiplied by 4 (not 2) and the cost should be $20.



3. The formulas for the areas of a square, rectangle, trapezoid, parallelogram and circle should now be elicited and changes made in the dimensions, as in the two previous problems. Students at first should be encouraged to substitute simple integers when problems are stated in terms of doubling, tripling, halving a dimension. Some examples follow.

4. The sides of a square are doubled. How is the area changed? How does the area change when the sides of the original square are tripled?
   Recall: $A = s^2$
   If $s_1 = 1$, $A_1 = 1$, $A_2 = 4$, $A_3 = 9$
   If $s_1 = x$, $A_1 = x^2$, $A_2 = (2x)^2 = 4x^2$, $A_3 = (3x)^2 = 9x^2$

5. How is the area of a trapezoid changed if:
   a) both bases are tripled, h remaning constant.
   b) h is tripled, the bases held the same.
   c) the bases and h are all tripled.

   $A = \frac{1}{2}h(b_1+b_2)$

   a) $A = \frac{1}{2}h(3b_1+3b_2) = 3(\frac{1}{2}h(b_1+b_2))$
      The area is tripled.
   b) $A = \frac{1}{2}(3h)(b_1+b_2) = 3(\frac{1}{2}h(b_1+b_2))$
      The area is tripled.
   c) $A = \frac{1}{2}(3h)(3b_1+3b_2) = 9(\frac{1}{2}h(b_1+b_2))$
      The area is multiplied by 9.

APPLICATIONS:

1. The above may also be applied to volume formulas such as V cyl. = $\pi r^2 h$, V cone = $\frac{1}{3}\pi r^2 h$, V sphere = $\frac{4}{3}\pi r^3$, V rect. solid = lwh, V cube = s^3.

2. Suppose, for some figure, $B = XYZ^2$. How will B change if:
   a) only X is doubled? (B is doubled)
   b) only X and Y are doubled? (B is quadrupled)
   c) only Z is multiplied by 4? (B is multiplied by the square of 4, i.e., 16)
   d) X, Y doubled, Z halved? ($B = (2X)(2Y)(\frac{Z}{2})^2 = XYZ^2$, i.e., B remains held the same)

SUMMARY:

1. What happens to the area of a figure when every dimension is doubled? (The new figure's area is four times that of the original.) How does the perimeter change? (It is doubled.)
2. When two similar figures have corresponding sides in the ratio 1:3, what is the ratio of their areas? (1:9) Their perimeters? (1:3)
3. Generalize these answers for similar figures with x:y as the ratio of similitude. (areas $x^2:y^2$, perimeters x:y)

HOMEWORK:

1. Using the formulas: 1) $A = bh$, 2) $A = \frac{1}{2}bh$, 3) $A = \pi r^2$, 4) $A = s^2$ and 5) $A = \frac{1}{2}h(b_1+b_2)$; state how A will change:
   a) in formulas 1) and 2) when b is tripled.
   b) in formula 3) when r is multiplied by 5.
   c) in formula 4) when s is divided by 3.
   d) in formula 5) when h, b_1, b_2 are all doubled.
2. If the length of each side of a square is tripled, how is its perimeter changed? How is its area changed?
3. If the radius of a circle is tripled, how is its cicumference affected?
4. The formula for the volume of a right circular cylinder is $V = \pi r^2 h$. How does the volume change if both r and h are tripled?
5. The formula for the volume of a sphere is $V = \frac{4}{3}\pi r^2$. If the radius, r, is tripled, how is the volume affected?
6. The formula for the area of the surface of a sphere is $S = 4\pi r^2$. If the radius is tripled, how is the surface area changed? Check your answer by letting r = 1" and r = 3".
7. The dimensions of a box are 2" x 3" x 4". How is the volume affected if each dimension is tripled?

8. A photographer "blows up" a picture to 5 times the original size.
   a) How are "corresponding" lengths affected by this enlargement?
   b) How are "corresponding" areas affected?
   c) How are "corresponding" volumes affected?

9. Suppose a faithful enlargement is made by an expert sculpture. If this enlargement is made by a factor of e:
   a) How are "corresponding" measurements of length affected?
   b) How are "corresponding" measurements of area affected?
   c) How are "corresponding" measurements of volume affected?

10. Suppose instead of enlarging, an accurate model is made so that we have a shrinkage by a factor of s. How are corresponding measurements of length, area and volume affected?

Lesson 48 - Coordinate Geometry (#1)

AIM: How do we use ordered pairs of numbers to locate points in the plane?

PERFORMANCE OBJECTIVES: The student will be able to...

1. Draw and label axes and quadrants of a coordinate plane.
2. Name the abscissa and ordinate of a point in the plane.
3. Graph given ordered pairs of numbers in a coordinate system.

VOCABULARY: quadrant, X-axis, Y-axis, origin, abscissa, ordinate, coordinate, ordered pair

MATERIALS: graph paper, graph stencil for chalkboard, straight edge

MOTIVATION:

Define these terms:
quadrilateral (a polygon with four sides)
quadruplet (one of four children born of one pregnancy)
quadruped (an animal (mammal) with four feet)

DEVELOPMENT:

1. Discuss motivation (review definitions). State: Today we will learn how to divide a plane into quadrants. Ask, "How many quadrants do you think will be formed?"
2. Have students draw a horizontal line near the center of the graph paper and a vertical line near the center of the paper. These are X and Y axes respectively, dividing the plane into four quadrants. We need some agreements, some vocabulary and some mathematical notation:
   The origin is the point of intersection of the X and Y axes.

From the origin, right and up are considered positive directions.

From the origin, down and left are considered negative directions.

Label both directions on each axis (+x, -x, +y, -y).

Label the quadrants (I, II, III, IV: starting in upper right, moving counterclockwise).

An ordered pair of numbers is used to identify the "address" of any point in the plane. The numbers are written within parentheses, separated by a comma: (x,y).

The "abscissa" is the first number of the ordered pair; it gives right/left information.

The "ordinate" is the second number of the ordered pair; it gives up/down information.

The "coordinates" are the abscissa and ordinate taken together.

We can write the ordered pair given any point and we can locate the point given any ordered pair of real numbers.

APPLICATIONS:

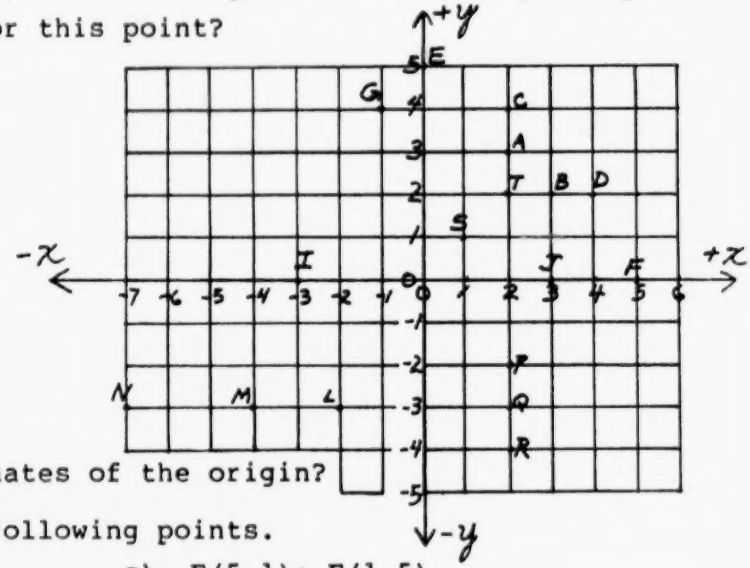
1. Locate some points, for example: (5,4) (-7,3) (2,3) (-3,5) (7,-2).
2. Does (5,-1) name the same point as (-1,5)? Explain your answer.
3. Plot some points on board using graph stencil. Ask students for coordinates, abscissas, ordinates.
4. What are the coordinates of the origin?
5. Where will you find all points which have positive abscissas? ("to the right of the y-axis") In which quadrant will a point lie if its abscissa and ordinate are both negative? What must be true of the coordinates of any point which lies in quadrant III?
6. Give the coordinates of any point which lies on the y-axis; in quadrant II; above the x-axis; exactly five units from the origin; etc.

SUMMARY:

1. What is an "ordered pair"?
2. How are points in the plane associated with ordered pairs of real numbers? (Every point in the plane can be associated with a unique ordered pair of numbers, and every ordered pair of numbers can be associated with a unique point.)
3. "Origin" means beginning or starting point (as in "original"). What activities "begin" at (0,0) to justify this special name for this point?

HOMEWORK:

1. Find the coordinates of: A,B,C,D, E,F,G,H,I,J, K,L,M,N,P,Q,R.



2. What are the coordinates of the origin?
3. Locate each of the following points.
   a) A(2,1); B(1,2)
   b) C(-3,2); D(2,-3)
   c) E(5,1); F(1,5)
   d) G(-5,1); H(1,-5)
   e) What seems to happen when the coordinates are interchanged?
4. Locate each of the following points.
   a) A(1,3); B(-1,-3)
   b) C(-1,4); D(1,-4)
   c) E(-3,-5); F(3,5)
   d) G(6,-2); H(-6,2)
   e) What seems to happen when the signs of the coordinates are changed?
5. If the product of the coordinates of a point is zero, where must the point be located?
6. If the product of the coordinates of a point is positive, where must the point be located? If the product is negative, where must the point be located?

7. If both coordinates are exactly the same, where must the point be located? (Try: (2,2), (3,3), (-4,-4).)

8. If both coordinates are different only in their signs, where must the point be located? (Try: (2,-2), (-2,2), (5,-5), (-5,5).)

9. If the center of a circle is located at the origin and one of its points is at (5,0), what is
   a) its area in terms of π?
   b) its circumference in terms of π?

10. Find the coordinates of the midpoint of the line segment $\overline{AB}$ if
   a) A is at (3,6) and B is at (5,1).
   b) A is at (-3,4) and B is at (3,-4).
   c) A is at (3,5) and B is at (5,3).

11. Write definitions of: 1) origin, 2) quadrant, 3) abscissa, 4) ordinate, 5) ordered pair, 6) coordinate of a point.

Lesson 49 - Coordinate Geometry (#2)

AIM: How do we use the coordinate plane to determine areas of geometric figures?

PERFORMANCE OBJECTIVES: The student will be able to...

1. Determine the length of a segment parallel to X or Y axis.
2. Find the perimeter and area of rectangles with sides parallel to the axes.
3. Find areas of right triangles with sides parallel to X and Y axes.

VOCABULARY: Review: perimeter of rectangle, area of a rectangle, length of a line segment

MATERIALS: graph paper, straight edge, rectangular graph stencil.

MOTIVATION:

1. Consider the points: A(-5,6), B(2,6). What is the length of the segment joining A to B?
2. Plot on graph paper; then name the figure formed by joining in order the points: (0,1), (7,1), (7,-3), (0,-3).

DEVELOPMENT:

1. Discuss motivation problem #1 with students. Conclude: For lines parallel to the X-axis, the length of the segment is the (absolute value of the) difference of the abscissas. (Show how a rectangle can be formed; the length of projection on the axis is the same as the length of the original line segment. This justifies using the difference.) Conclude the analagous principle for lines parallel to the Y-axis.

2. Discuss motivation problem #2. The figure is a rectangle. The area can be determined by using dimensions and the formula, $A = l \cdot w$, or by counting squares. Area is in square units, length is in units of measure.

APPLICATIONS:

1. State whether each set of points are the vertices of a rectangle. If so, find dimensions and area:
   a) (-3,5), (4,-2), (-3,-2), (4,5)
   b) (0,0), (0,5), (5,5), (5,0)
   c) (-4,0), (-2,5), (1,5), (-1,0)
2. Draw triangle. Then find its area.
   a) (6,1), (8,1), (8,4)
   b) (-4,10), (-4,2), (4,10)

SUMMARY:

Areas of rectangles and triangles with sides parallel to X and Y axes can be calculated by "counting squares" or by using the formulas, $A = lw$ and $A = \frac{1}{2}bh$. Which approach do you prefer? Explain.

HOMEWORK:

1. The ends of a diameter of a circle are located at (-1,0) and (5,0). In terms of π express:
   a) its area.
   b) its circumference.
   c) What is the area of a square circumscribing the circle? (Its sides touch the circle.)
2. Do problem 1 where the ends are located at (-2,0) and (10,0). How do the answers compare?
3. What is the area of the square with vertices at: (7,0), (0,7), (-7,0), (0,-7)?
4. What is the area of the triangle with vertices at:
   a) (0,0), (0,4), (7,0)
   b) (0,0), (-1,5), (5,5)
   c) (0,2), (-1,5), (5,5)

5. In terms of π find the area and perimeter of the semicircle whose diameter has endpoints located at:

   a) (0,0), (0,12)

   b) (-6,0), (0,6)

   c) (0,8), (0,2)

Lesson 50 - Coordinate Geometry (#3)

AIM: How do we to graph linear equations in two variables?

PERFORMANCE OBJECTIVES: The student will be able to...

1. Evaluate open sentences in two variables to determine whether given ordered pairs are elements of the solution set. (Determine if a point lies on the line.)
2. Determine three ordered pairs in the solution set of a given linear equation.
3. Use the coordinate plane to draw the graph of the line that represents all solutions to a given linear equation.
4. State why the equation is called linear.

VOCABULARY: collinear, noncollinear, replacement set, linear equation

MATERIALS: graph paper, graph stencil for chalkboard, yardstick (optional), overhead projector for transparencies

MOTIVATION:

Solve the equations:

1. $2x + 5 = -3$ 2. $y = x + 7$

DEVELOPMENT:

1. Discuss motivation problems. The <u>first</u> equation is a review: An open sentence in one variable (usually) yields one element in its solution set. The <u>second</u> equation (with two variables) may have <u>many</u> elements in its solution set. We seek ordered pairs to make a true statement. Suppose $x \in \{0,1,2,3,4,5,6,7,8,9\}$
   $y \in \{0,1,2,3,4,5,6,7,8,9\}$
   Then solution set is $\{(0,7), (1,8), (2, 9)\}$. However, if replacement set is the set of real numbers, then the solution set has an <u>infinite</u> number of ordered pairs.

2. How is all this related to the coordinate plane? Plot the points which correspond to the ordered pairs (0,7), (1,8), (2,9). Elicit other ordered pairs (x,y) which satisfy $y = x + 7$, such as (3,10), (-1,6), (2,5). (It is easy to calculate y values which correspond to chosen x values, because of the form of the equation.) Students should observe that all of the points which correspond to solution-pairs for this equation lie along a straight line. Identify the coordinates of some points which lie on the same line, between those already graphed (e.g., $(1\frac{1}{2},8\frac{1}{2})$). When $x = 1\frac{1}{2}$ and $y = 8\frac{1}{2}$, is the equation a true statement? Challenge the students to find a point on this line which does not satisfy the equation or a pair of values which makes the equation true. Then examine a point which is off the line. Conclude that the line and the set of all solution-pairs are the same.

3. Consider the equation $2x + y = 7$. (Have students copy the following procedure as it is elicited from the class.)

Procedure to graph a linear equation:

		Result
1.	Solve for y in terms of x.	$y = -2x + 7$
2.	Find at least three ordered pairs in the solution set by choosing values for x. (Keep x values small, choose 1 positive, 1 negative, and 0.)	(-2,?) (0,?) (3,?)
3.	Calculate the y-value for each chosen x-value.	(-2,11),(0,7) (3,1)
4.	Plot points represented by the ordered pairs.	
5.	Use a straightedge to draw the line through the collinear points. If the points are not collinear, check the work.	
6.	Label the line with the original equation.	
7.	Check all labels: x-axis, y-axis, origin, line.	

Ask, "Why are <u>three</u> points located before this line is drawn?" (Two points are sufficient to determine the location of the line; the third point is for "insurance". If they do not appear collinear, recheck your calculations to determine which point is misplaced.)

APPLICATIONS:

1. Tell whether the given ordered pair is a solution of the open sentence:
   a) $2x + y = 11$ (3,5) c) $x + y = 6$ (-3,-3)
   b) $x - y > 11$ (7,-8) d) $4x + 3y = 16$ (4,0)
2. Use the procedure above to graph $x - y = 8$.

SUMMARY:

1. How would you determine whether a given point lies on a line? (See if the x and y values of its location satisfy the equation of the line.)
2. Give examples of equations which produce lines when graphed.
3. Describe the procedure you would employ to produce the graph of the solution set for a linear equation.

HOMEWORK:

1. a) Find 3 points whose coordinates satisfy the equation $y = x + 1$.
   b) Find the coordinates of a fourth point on the line containing the 3 points in a). Do the coordinates satisfy the equation $y = x + 1$?
2. a) Find 3 points whose coordinates satisfy the equation $x + y = 10$.
   b) Draw the line containing these points.
   c) Find a point on this line whose coordinates are not integers. Check whether its coordinates satisfy $x + y = 10$.
3. For the equation $2x + 3y = 11$, find the corresponding values of x or y that satisfy this equation in the table:

x	y
1	?
?	1
0	?
?	0

Plot and draw a straight line through these points.

4. Indicate by writing a, b, n whether the point has coordinates satisfying equation a or b or neither.

   Equation a): $3x - y = 8$

   Equation b): $x + 2y = 5$

   1) (7,13)
   2) (7,-1)
   3) (3,1)
   4) (0,8)
   5) $(0,2\frac{1}{2})$
   6) (-5,5)

5. Find some points whose coordinates satisfy $y > x + 1$. Try to describe where all such points must be located.

6. a) Find the line for each of the following equations:

   $2x + y = 11$ $\qquad$ $x + y = 7$

   b) Find the coordinates of the point where the lines cross.

   c) What is the significance of this point?

7. Challenge:

   a) Graph the line whose ordinate is 3 less than twice its abscissa.

   b) State which of the 3 points named lies on the graph of the given equation:

      a) $3x + 2y = 12$; (0,6), (4,1), $(\frac{1}{3},\frac{1}{2})$

      b) $8x - y = 3$; $(\frac{1}{2},11)$, (-0.5,-7), $(\frac{1}{8},2)$

Lesson 51 - Coordinate Geometry (#4)

AIM: How do we use the coordinate plane to graph lines parallel to x and y axes?

PERFORMANCE OBJECTIVES: The student will be able to...

1. Graph equations of the form $x = a$, $y = b$.
2. Use x- and y-intercepts to graph lines in the coordinate plane.
3. Write the equation of a line drawn parallel to either axis.

VOCABULARY: x-intercept, y-intercept

MOTIVATION:

1. Name the coordinates, then graph eight points, each of which has abscissa equal to 3.
2. Consider the equation $y = -2$. Can you draw its graph?

DEVELOPMENT:

1. Discuss motivation problems.
   If the abscissa is always 3, then some points are (3,5), (3,2), (3,0), (3,-1), etc. The graph of this line is the graph of $x = 3$, a line parallel to y-axis and three units to the right.
   $y = -2$ will have points (3,-2), (7,-2), (0,-2), (-1,-2), etc. The graph is a line parallel to the x-axis and two units below.
   Ask: "How are these cases related?" (In each, an equation of the form variable = constant produces a line parallel to the axis.)
2. Note that the graph of $ax + by = c$, where $a \neq 0$ and $b \neq 0$, will not be parallel to either axis. We will now consider the points where these lines intersect the x-axis and the y-axis. Use yesterday's procedure to graph the line whose equation is $3x + 2y = 12$. After students have worked on the graph for a few minutes, ask, "How

many chose the x-value of 0? (All should have.) How many chose the x-value of 4? (Some may have.) What are corresponding y-values for each? (0,6), (4,0) These are important points, the "intercepts" of the line. 4 is known as the x-intercept. (It is the x-value of the point where the graph of the line crosses the x-axis.) Similarly, 6 is the y-intercept. In general, coordinates of any point on the x-axis are of the form (x,0); coordinates of any points on the y-axis are of the form (0,y).

Define: the abscissa of the point where the line intersects the x-axis is called the x-intercept of the line.

Define: the ordinate of the point where the line intersects the y-axis is called the y-intercept of the line.

Emphasize that an intercept is a number (not an ordered pair). The use of intercepts to draw a graph of a linear equation is simple and convenient. It is easy to find these values by substituting zero for each variable in turn. Here we plot (0,6) and (4,0) to sketch $3x + 2y = 12$.

APPLICATIONS:

1. Identify the x- and y-intercepts of:
   a) $2x + y = -4$ b) $3x - 2y = 6$
2. Graph the line whose equation is $y = 4$.
3. Write the equation of the line parallel to the y-axis and 3 units to the left.
4. Graph $y = 0$, $x = 1$, $x = -5$ and give a verbal description of each.
5. Use x- and y-intercepts to sketch the graphs of:
   a) $2x + y = 6$ b) $4x + 2y = -8$
   c) $x - 3y = 3$
6. What is the equation for the y-axis? ...the x-axis?

SUMMARY:

1. Describe the graph of an equation of the form $x = a$. (Line $x = a$ is parallel to the y-axis and a units to the right (or left) of the y-axis.)
2. Describe the graph of an equation of the form $y = b$. (Line $y = b$ is parallel to the x-axis and b units above (or below) the x-axis.)
3. How many x- and y-intercepts can be used to sketch the graph of a straight line of the form $ax + by = c$?
4. Given an example of an equation, which will produce a vertical line passing through quadrants II and IV?

HOMEWORK:

1. Graph:
   a) $x = 3$ b) $y = 5$

   Find the coordinates of their point of intersection.
2. For the graph of $2x - 3y = 12$, what are the coordinates of the point where the graph crosses:
   a) the x-axis? b) the y-axis?
3. What is the equation of the line parallel to:
   a) the x-axis passing through (5,7).
   b) the y-axis passing through (5,7).
4. For the graph of $\frac{x}{2} + \frac{y}{3} = 1$, what are the coordinates of the point where the graph crosses:
   a) the x-axis? b) the y-axis?
   c) Try to generalize to $\frac{x}{a} + \frac{y}{b} = 1$, $ab \neq 0$.
5. A point moves so that its ordinate is always one more than its abscissa.
   a) Write an equation for this line.
   b) Sketch its graph.
6. Challenge Exercises:
   a) Use intercept method to draw graphs of:
      a) $y = 2x + 6$ b) $3x - 4y = 8$
      c) $3x - y = 0$
   b) List some difficulties associated with the use of the intercept method for graphing straight lines.

Lesson 52 - Coordinate Geometry (#5)

AIM: How do we find the slope of a line?

PERFORMANCE OBJECTIVES: The student will be able to...

1. Define a slope as a number which tells how a line is slanted.
2. State how lines with positive, negative, zero and undefined slopes are inclined.
3. State the two point slope formula: $M = \frac{y_2-y_1}{x_2-x_1}$
4. Find the slope of any nonvertical line, given the coordinates of two points on the line.

VOCABULARY: slope, ratio, delta y (rise), delta x (run)

MATERIALS: graph paper, graph stencil, meter stick

MOTIVATION:

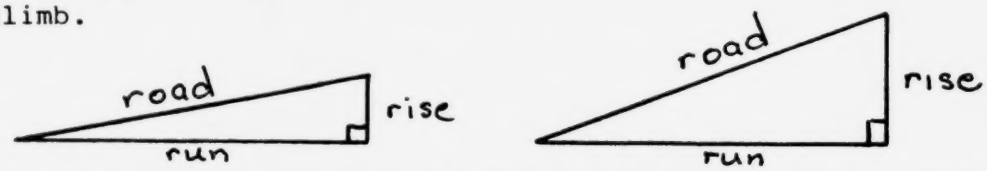
1. Plot the points: A(2,3), B(7,6), C(7,3), D(12,5).
2. Draw $\overline{AB}$ and $\overline{CD}$.
3. Which segment is longer?
4. In what other way could these segments be compared?

DEVELOPMENT:

1. Discuss motivation. To compare the lengths of the segments, vertical and horizontal segments may be added to the graph so that $\overline{AB}$ and $\overline{CD}$ appear as hypotenuses of right triangles. Elicit that each right triangle has one leg (horizontal) which is 5 units long. The vertical legs are not the same length. Even without applying the Pythagorean theorem, it should be obvious that $\overline{AB}$ is longer than $\overline{CD}$ (the exact lengths are $\sqrt{35}$, about 5.8, and $\sqrt{29}$, about 5.4).

The students should also observe that there is a similarity and a difference in the orientation of these segments. Elicit: They are not parallel; both "go up toward the right"; $\overline{AB}$ "rises faster", "goes up more", "is steeper"; $\overline{CD}$ "is flatter", "is less of a slope". Students should use these or similar phrases to express their intuitive sense of the steepness or slope of the lines of which these segments are a part. Extend the segments and elicit: steepness or slope is independent of length.

2. When planning roads for trains or autos, or ramps for handicapped, engineers are careful to avoid too steep a climb.



In the figures, vertical distances climbed are equal, but the road on the right is much steeper. How will we measure the steepness, or slope of the line? Using coordinates from Motivation problem 1 above, we have a line passing through A(2,3) and B(7,6). Between A and B, the line rises a distance of 3 units covering a horizontal distance of 5 units. The line passing through C and D rises a distance of 2 units covering a horizontal distance of 5 units.

We define the slope of a line as follows: Slope of a line is a number which expresses the ratio $\frac{\text{rise}}{\text{run}}$.

Slope of a line = $\frac{\text{change in vertical distance}}{\text{change in horizontal distance}}$.

Thus, the slope of $\overline{AB}$ is 3/5, a number; the slope of $\overline{CD}$ is $\frac{2}{5}$.

The teacher should take any two points on either line and demonstrate that the slope of the line is characteristic of the orientation of the line, and not a result which changes according to the points selected. In lowest terms, the ratios taken for the same line are identical. The slope of a line is the number we assign to a line (segment) that tells about its direction.

3. In the coordinate plane, slope is the difference in y-values (y) divided by difference in x values (x).

The letter usually used to represent slope is m.

Formula: $m = \frac{y_2-y_1}{x_2-x_1}$ for given points (x_1,y_1) and (x_2,y_2).

For example, given (-2,7) and (3,-1), the slope is $\frac{-1-7}{3-(-2)} = \frac{-8}{5}$.

4. Sketch, then determine the slope of the line joining:

   a) (0,2) to (2,5)
   b) (11,3) to (1,1)
   c) (3,2) to (3,-1)
   d) (-2,-1) to (5,7)
   e) (-2,2) to (3,2)

5. Elicit:

   a) Reading from left to right, a rising line indicates positive slope;
   b) Reading from left to right, a falling line indicates negative slope;
   c) a horizontal line has zero slope;
   d) a vertical line tells us that the slope is undefined (the horizontal change is zero, and division by zero is undefined);
   e) the greater the absolute value of the slope, the closer to vertical the line will be;
   f) the smaller the absolute value of the slope, the closer to horizontal the line will be.

APPLICATIONS:

1. Give students pairs of points and ask them to calculate the slope of the line through the points.
2. Present segments and ask them to tell by inspection whether the slope is positive, negative, zero, or undefined.
3. Present two segments and ask them to tell which has the greater slope.
4. Ask students to sketch a line (or hold a yardstick) to show a line with slope zero; +1, +5, $+\frac{1}{2}$, -1, $-\frac{1}{4}$, -10.

SUMMARY:

1. What is the slope of a line?
2. What is the significance of the sign of the slope?
3. What is the significance of the absolute value of the slope?
4. How is the slope of a line calculated?
5. What real life situations provoke concern about slope?
6. What do you think will be true of the slope of each of two <u>parallel</u> lines?

HOMEWORK:

1. For the line whose equation is $y = 2x - 3$:
   a) find the missing coordinates: A(2,?), B(3,?), C(4,?).
   b) find the slopes for: 1) $\overline{AB}$, 2) $\overline{BC}$, 3) $\overline{AC}$.
   c) What seems to be the case for the slope when two points are selected?
   d) Try to generalize.
2. For the line whose equation is $y = 2x + 3$, do example 1.
3. Do example 1 for $y = 2x$.
4. What happens to the formula $m = \frac{y_2-y_2}{x_2-x_1}$ when:
   a) The line is parallel to the x-axis?
   b) The line is parallel to the y-axis?
5. Write the equation of a line having a slope of 3. Check your answer by selecting 2 points on this line and using the slope formula $m = \frac{y_2-y_1}{x_2-x_1}$.
6. <u>Challenge exercises</u>: Cite 5 examples in athletics where slope can be a major factor in an athlete's strategy (ski slope, auto racing, running marathon, golf (club selection), field sports (drainage)).

Lesson 53 - Coordinate Geometry (#6)

AIM: How do we identify the slope and y-intercept of a straight line from its equation?

PERFORMANCE OBJECTIVES: The student will be able to...

1. Find the slope and y-intercept of a line from its given equation.
2. Write the generalized slope - intercept form of the equation of a straight line: $y = mx + b$.

VOCABULARY: slope - intercept form of equation of a straight line, coefficient, constant

MATERIALS: graph stencil, meter stick, graph paper, overhead projector, teacher-prepared transparencies (if desired for transparency lesson)

MOTIVATION:

Use graph paper (with a separate set of axes for each) to draw the graphs of:

1. $y = 2x - 3$ 2. $y = -x + 3$ 3. $y = \frac{3}{2}x + 2$

(Note: students should use a table of values, the procedure of lesson #50, to construct the graphs of the given equations.)

After graphing, calculate the slope of each line.

DEVELOPMENT:

1. Show graphs (on large grids) of each equation from motivation exercises. Ask students to state the slope of each line by counting the horizontal change from one lattice point to another, then by counting vertical change. Slope is the ratio of vertical change to horizontal change, or $\frac{\Delta y}{\Delta x}$.
   Have students identify the y-intercept (the ordinate of point where the line crosses the y-axis).

Have students complete the chart shown below.

EQUATION	SLOPE = $\frac{y}{x}$	y-INTERCEPT
$y = 2x - 3$	$\frac{2}{1}$	-3
$y = -x + 3$	$-\frac{1}{1}$	+3
$y = \frac{3}{2}x + 2$	$\frac{3}{2}$	+2
Try to elicit generalization:		
$y = mx + b$	m	b

Note that equation must be solved for y in terms of x.

APPLICATIONS:

Identify the slope and y-intercept for each equation:

1. $y = 3x - 1$
2. $y = \frac{x}{2}$
3. $3y = -2x - 9$
4. $2y - 5x = 4$
5. $5y = 3x - 10$

*6. $ax + by = c$ (See challenge exercise in Homework.)

SUMMARY:

1. How are the values of m and b in the equation $y = mx + b$ significant for the orientation of the line? (If $y = mx + b$, then m, coefficient of x, tells us the slope of the line and b, the constant, is the y-intercept.)
2. What is the meaning of "y-intercept"? (The ordinate of the point where the line crosses the y-axis.)

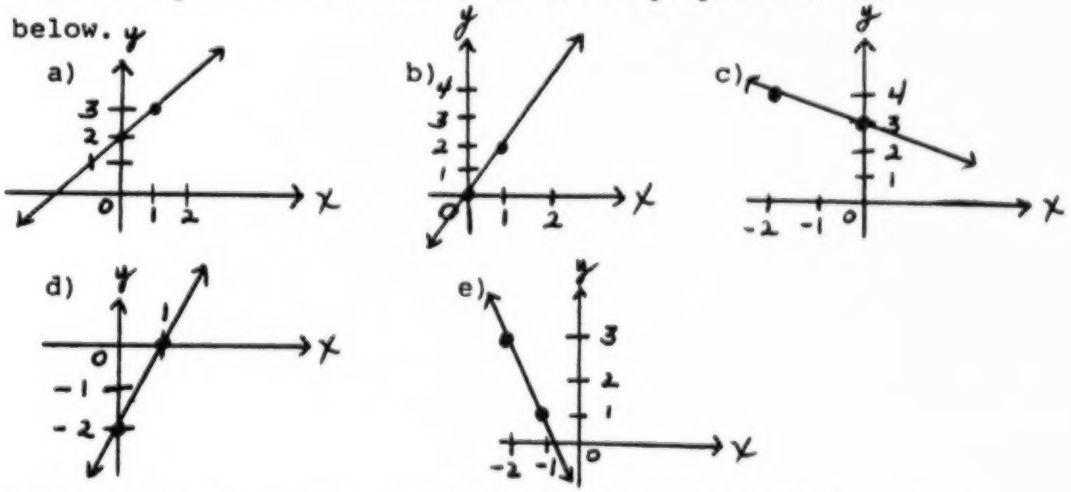
HOMEWORK:

1. For each of the following linear equations, find the slope two ways.
   a) by the formula $m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{\Delta y}{\Delta x}$.
   b) by expressing the equation in the form $y = mx + b$.
      a) $2y = 4x + 1$
      b) $2y = -6x + 3$
      c) $x + y = 5$
      d) $2x - y = 6$
      e) $\frac{x}{2} + \frac{y}{3} = 6$

2. For each of the equations in example 1, find the coordinates of the point of intersection of the line with
   a) the x-axis. b) the y-axis.

3. Find an equation of the line whose slope (m) and y-intercept (b) are given. Then draw its graph.
   a) $m = 3, b = -1$ c) $m = -2, b = 1$
   b) $m = -1, b = 2$ d) $m = \frac{1}{2}, b = 4$

4. Find an equation for each line whose graph is shown below.



5. What seems to be the case for all lines with the same slope?

6. Find an equation for each of the following lines with intercepts given:

	x-intercept	y-intercept
a)	5	5
b)	5	-5
c)	-5	5
d)	-5	-5
e)	2	3

7. Challenge Question: What is the slope-intercept form for the general equation $ax + by = c$?
   Ans: $y = \frac{-a}{b}x + \frac{c}{b}$. That is, $m = \frac{-a}{b}$, $b = \frac{c}{b}$.

Lesson 54 - Coordinate Geometry (#7)

AIM: How do we graph a linear equation using the slope-intercept method?

PERFORMANCE OBJECTIVES: The student will be able to...

1. Graph linear equations of the form $y = mx + b$.
2. Graph linear equations of the form $ax + by = c$.
3. Write the equation of a straight line given its slope and y-intercept.

VOCABULARY:

slope intercept method for graphing a linear equation

MOTIVATION:

1. Write an equation of the line having slope -2 and passing through the point (0,3).
2. Solve for y in terms of x: $2x + 3y = 9$.

Can you give a good reason for performing exercise 2?

DEVELOPMENT:

1. Exercise 1 should result in students writing $y= -2x + 3$. They may need to be reminded that 3 is the y-intercept. Exercise 2 produces the equation $y = -\frac{2}{3}x + 3$. Elicit that the slope-intercept method is the most preferred method for graphing straight lines because it is efficient for quick sketches and for more detailed constructions. Having the equation in this form will make it easier to graph.
2. Elicit and apply the "procedure for graphing a straight line in two variables by the slope-intercept method".

STEP	PROCEDURE	EXAMPLE
1.	Solve for y in terms of x, being sure to write the equation in the form $y = mx + b$.	$3x - 2y = 6$ $y = \frac{3}{2}x - 3$
2.	Plot the point (0,b)	Plot (0,-3).
3.	Count from (0, b) squares as indicated by the ratio $\frac{y}{x}$ to establish the slope.	Count 2 units to <u>right</u> and 3 units <u>up</u> from the point (0, -3)
4.	Connect two points* (and beyond) with straight edge.	Connect (0,-3) to (2,0).
5.	Label the line with its original equation.	$3x - 2y = 6$

*Note: Using the slope, one can <u>quickly</u> locate several <u>more</u> points for a neater line. Also, since $\frac{+3}{+2} = \frac{-3}{-2} = +\frac{3}{2}$, one can locate more points to the left of the axis by counting squares "down 3 and left 2".

APPLICATIONS:

Graph each line whose equation is given:

1. $y = \frac{3}{5}x + 2$

2. $3x - y = -1$

3. $2x + 3y = 12$

Note: In this equation the slope $-\frac{2}{3}$ may be interpreted as $\frac{+2}{-3}$ (up 2, left 3) or as $\frac{-2}{+3}$ (down 2, right 3).

4. $y = 2x + 4$

5. $y = 2x + 1$

From problems 4 and 5, elicit: If two lines have the same slope, they are parallel, and conversely.

SUMMARY:

1. Describe the table of values method for graphing a linear equation.
2. Describe the slope-intercept method.
3. What are the advantages of the slope-intercept method?

HOMEWORK:

1. Use the slope-intercept method for graphing:

   a) $y = 2x - 3$ d) $2x - y = 1$

   b) $y = -2x + 3$ e) $3x + 2y = 6$

   c) $2x + y = 1$ f) $-3x + 2y = 6$

2. Graph each of the following by finding the x- and y-intercepts.

   a) $3x + 2y = 6$ d) $2y = 3x + 6$

   b) $x + 3y = 6$ e) $3y = 6 - 2x$

   c) $2x - y = 4$ f) $-3y = 6 - 2x$

3. Find the area of the right triangle cut off in the first quadrant by the line for:

   a) $4x + y = 8$ d) $3x + 4y = 6$

   b) $x + 4y = 8$ e) $x + y = 7$

   c) $3x + 4y = 12$ f) $y = 7 - 2x$

4. Find the coordinates of the point of intersection of the lines for:

   a) $y = 2x + 1$ and $y = x + 4$.

   b) $x + y = 10$ and $x - y = 2$

   c) $2x + 3y = 12$ and $y = 5 - x$

   d) $2y = 3x + 1$ and $x + 2y = 1$

5. Which of the following have graphs that are parallel lines?

   a) $y = -2x + 1$ c) $2x + y = 5$

   b) $x + 2y = 1$

6. <u>Challenge Exercise</u>: Write an equation of the straight line whose slope is 2 and passes through the point $(3,-1)$. $(y = 2x - 7)$

Lesson 55 - Coordinate Geometry (#8)

AIM: How do we solve a system of two linear equations by using graphical methods?

PERFORMANCE OBJECTIVES: The student will be able to...

1. Define the term "system of simultaneous linear equations".
2. Determine the solution set of a system of two linear equations by graphing.
3. Check the solution.
4. State conditions under which no real solution exists.

VOCABULARY: system of linear equations in two variables, consistent, inconsistent, dependent, coincident lines

MATERIALS: Teacher-prepared viewgraph stencils and overhead projector are recommended. This is a lengthy lesson.

MOTIVATION:

Graph on the same set of coordinate axes:

$y - x = -2$ $\qquad\qquad$ $2x + y = 4$

DEVELOPMENT:

1. Discuss the concept: All ordered pairs that satisfy the equation $y - x = -2$ lie on the line that is the graph of $y - x = -2$. All ordered pairs that satisfy $2x + y = 4$ lie on the line that is the graph of $2x + y = 4$. When two linear equations are placed on the same set of coordinate axes, we have a SYSTEM OF SIMULTANEOUS LINEAR EQUATIONS. More generally, a system of equations is any set of equations being considered for a <u>common</u> solution. Can you specify a point that lies on both lines? This point (if such a point exists) satisfies both equations simultaneously, and is the solution set of the system.

Solution set is written: $\{(2,0)\}$. (The set whose element is an ordered pair.)

2. The solution is the pair of coordinates of the point where the lines intersect. How can this solution be checked? How do we know that no other solutions exist? Can you suggest a situation where there is no solution? Elicit: a system of parallel lines. (For example: $y = 2x + 3$, $2x - y = 1$.) How can this answer (no solution) be checked?

3. There is one more possibility. Can you suggest a situation where there is more than one element in the solution set? Try $2x - y = 3$, $2y = 4x - 6$.

4. For all systems, the correct format should be stressed: Label axes, origin, each line, solution set and check.

APPLICATIONS: Solve:

1. $x + 2y = 7$
   $y = 2x + 1$
   $\{(1,3)\}$

2. $2x + y = 9$
   $y = -2x + 11$
   $\emptyset$ or $\{\ \}$ (empty set)

3. $2x - y = 5$
   $3y = 6x - 15$
   dependent system

SUMMARY:

What are the possible outcomes when the graphs of two linear equations are graphed in the same coordinate plane? Summarize responses in a chart. (Students should copy this chart into their notebooks.)

Systems of Simultaneous Linear Equations

Type of System	Graph	Number of Solutions
Consistent	Intersecting lines	one (ordered pair at intersection)
Inconsistent	Parallel Lines	none
Dependent	Coincident lines (one line on top of the other)	infinitely many

HOMEWORK:

1. Graph each pair of equations and decide whether the system is consistent (in which case specify the common solution set), inconsistent, or dependent.

   a) $x + y = -2$
      $x + y = 0$

   b) $y = 3x - 5$
      $2x + y = 5$

   c) $2y = x - 1$
      $2x - 4y = 2$

2. Find the area of the triangle bounded by the x axis and the intersecting lines:

   a) $x + 2y = 10$
      $y = x - 4$

   b) $2x - 3y = 5$
      $x + 3y = 7$

3. Graph each pair of equations and the equation obtained by adding the pair. Find their common solution.

   a) $x + y = 7$
      $x - y = 3$

   b) $2x + y = 5$
      $-2x + 3y = -1$

   c) $3x - 2y = 5$
      $x + 2y = -1$

   d) $-3x - 2y = -1$
      $3x + y = 2$

4. Challenge Exercise: Graph the system of equations.

   1) $2x - y = 5$

   2) $2x + 3y = 1$

   On the same set of axes graph $4x + 2y = 6$ (the sum of 1 and 2). What conclusion do you reach about the sum of the equations in a system?

Lesson 56 - Coordinate Geometry (#9)

AIM: How do we graph a linear inequality?

PERFORMANCE OBJECTIVES: The student will be able to...

1. Graph a linear inequality in two variables.
2. State the existence of a boundary line for the solution.
3. Specify that the solution is a region.
4. State the conditions under which the points on the boundary line will be included in the solution set.
5. Check one element from solution set in the original inequality.

VOCABULARY: plane divider, region, linear inequality

MOTIVATION:

1. Use the slope-intercept method to graph the line: $y = \frac{2}{3}x - 2$.
2. On a number line, graph $\{x: x > 3\}$.

DEVELOPMENT:

1. We are familiar with the concept of inequalities and with the coordinate system. Let us now combine the two ideas to graph a linear inequality using the Cartesian coordinate system.
   Suppose a line is drawn in a plane. Into how many sets of points is the plane divided? Develop the notion that the plane is divided into three sets of points: the half-plane on one side of the boundary line, the half-plane on the other side of the boundary line, and the boundary line itself.
   Consider the set of points: $\{(x,y): y \leq \frac{2}{3}x - 2\}$. In other words, we are looking for all ordered pairs (x,y) that satisfy $y \leq \frac{2}{3}x - 2$. Will the points on the line satisfy this inequality? Are there any other points in the plane whose coordinates will make the statement true?

Elicit: All points which make the statement true lie on the same side of the boundary line.

2. Elicit the Procedure for Graphing a Linear Inequality:
   1) Replace the inequality with an equal sign. $y = \frac{2}{3}x - 2$. This is the equation of the boundary line. Graph it.
   2) In this case, the boundary line will be solid since our boundary line is included in the solution set. (If boundary line is not included, we use a dashed line.)
   3) Select one point which does not lie on the boundary. Determine whether it makes the inequality true or false.
   4) Shade one half-plane (if the selected point made the inequality true, shade the half-plane which contains it; otherwise, shade the other half-plane).

   Ask, "What strategy should you use when selecting the 'test point'?" (Select a point with small integer coefficients or use (0,0).)

APPLICATIONS:

Graph:

1. $\{(x,y) : y < -2\}$ (dashed)
2. $\{(x,y) : y + 3x \leq 5\}$ (solid)
3. $\{(x,y) : 2x - y \leq 5\}$ (solid)
4. $\{(x,y) : 2x - y < -2\}$ (dashed)
5. $\{(x,y) : x > 3\}$ (dashed)
6. $\{(x,y) : -2 < x\}$ (dashed)

Note: Examples 5 and 6 illustrate why it is dangerous to rely on the direction of the inequality symbol to determine which half-plane to shade.

SUMMARY:

1. How do you know whether the solution set of a math sentence will be a line or a region?

2. How do you determine whether the boundary is included in the solution region or not?

3. What is the procedure to follow when graphing the solution set of an inequality?

HOMEWORK:

1. Graph each of the following. (Remember to use a dashed or solid line when appropriate.)

a)	$y > x$	e)	$\{(x,y): y - 2x > -1\}$
b)	$y \geq x - 2$	f)	$\{(x,y): y + 2x > -1\}$
c)	$x + y > 2$	g)	$\{(x,y): x + 2y < 4\}$
d)	$x + y \geq 3$	h)	$\{(x,y): x - 2y < 4\}$

2. Graph the common solution set for these systems.

a)	$y \geq 0$ $y > x - 2$	d)	$x < 1$ $x + y > 2$
b)	$y < 0$ $y > x - 2$	e)	$-2x > 5$ $2x + y \geq 4$
c)	$x \geq 0$ $x + y < 6$	f)	$x + y \leq 10$ $x - y > 1$

3. Graph the common solution set for these systems.

a)	$x \geq 1$ $x < 3$	d)	$x + y > 3$ $y = 2x$
b)	$y \geq 5$ $y < 3$	e)	$x + y = 3$ $2x + y \geq 2x$
c)	$x \geq 0$ $x + y < 6$	f)	$x + y \leq 10$ $x - y > 1$

4. <u>Challenge exercise</u>: Graph on the same set of axes:

$3x + y \geq 2$ $\qquad\qquad$ $2x - y < 5$

What is your conclusion about the solution set of these inequalities?

Lesson 57 - Coordinate Geometry (#10)

AIM: How do we use the coordinate system to find the solution set of a system of inequalities?

PERFORMANCE OBJECTIVES: The student will be able to...

1. Represent graphically the solution set of a system of two inequalities.
2. Choose ordered pairs in the solution set of the system.
3. Extend concept to include system of three inequalities.

VOCABULARY: region (review), parameter, compound open sentence

MOTIVATION:

Graph on the same set of axes $y < 2x + 3$ and $x + y \geq 1$. (Use the slope-intercept method to graph boundary lines.)

DEVELOPMENT:

1. How would you determine whether a particular ordered pair satisfies both inequalities simultaneously? (conjunction of conditions) How would you locate a point that satisfies the second inequality, but not the first? How would you specify a point that satisfies neither inequality?
   This topic is analogous to the solution of system of linear equations. Elicit the correct use of dashed and solid boundary lines, and correct labels for boundary lines and regions. The use of capital letters for regions is suggested.
2. Explain that in more advanced mathmatical work, (calculus, operations research, applications such as economics), we are interested in limits, profitability, production parameters, etc. One basic idea involves use of 3 inequalities in two variables. Consider, for example: $x + y > 2$
   $x + 2y \leq 8$
   $y \geq -2x + 4$

The region for their common solutions is a triangle that sets parameters for operation. Consider other possibilities with three inequalities in three variables.

APPLICATIONS:

Graph the following systems of inequalities.

1. $y \leq 5$
   $y > x + 3$

2. $2x + y \leq 4$
   $x + y > 5$

3. $2x + 3y \leq 6$
   $3x - 2y \geq 12$

4. $y > x$
   $y \leq 4$
   $x + y > 3$

SUMMARY:

Elicit summary from students: To graph a system of inequalities:

1. Graph and label boundary lines for each inequality (dashed or solid lines as indicated by the inequality symbol).
2. Shade the appropriate region for each (determined by a test point).
3. Choose a point in the doubly shaded region to check your solution. The solution region (the intersection of the solution regions for the separate sentences) should be labeled with a capitol letter.

HOMEWORK:

1. Graph the common solution set for these systems:

a) $y \geq 0$
   $y \leq 5$
   $x > 2$

b) $y \geq 2x - 1$
   $x \geq 0$
   $y > 10$

c) $y > 0$
   $y < 5$
   $y > 2x - 3$

d) $x + y < 10$
   $y \geq 0$
   $x > -1$

e) $x < 2$
   $x \geq 4$
   $y > 1$

f) $y = 2x - 2$
   $x \geq -3$
   $x < 5$

2. Graph the solutions for these conditions.

*a) $(y > 0) \wedge (x \geq 0)$

b) $(y > 0) \vee (x \geq 0)$

c) $(x + y < 10) \vee (x \geq 1)$

d) $(x + y < 10) \vee (x \geq 1)$

*e) $(x + y < 10) \wedge (x > 0)$

3. <u>Challenge exercise</u>: Graph the solution for the system of inequalities:

$$y < -\frac{1}{5}x + 6$$

$$y \geq x + 4$$

$$y \geq -x + 2$$

What kind of triangle is formed by this region?

Lesson 58 - Coordinate Geometry (#11)

AIM: How do we use algebraic methods to determine the solution set of a system of simultaneous linear equations?

PERFORMANCE OBJECTIVES: The student will be able to...

1. Solve systems of linear equations in two variables by the method of elimination by addition. (Restricted to cases where one of the variables drops out immediately.)
2. Check the solution by replacing variables in the original equations.

VOCABULARY: method of elimination by addition, review: addition property of equality, lattice point

MOTIVATION:

Solve graphically and check: $4x + y = 7$; $2x - y = 8$.

DEVELOPMENT:

1. The graphical solution is $\{(+2.5,-3)\}$ for this system of equations. Ask students if they felt something was wrong because the ordered pair did not coincide with a lattice point on the graph paper.
   Discuss how a system whose solution set is $\{(73, 197)\}$ might be shown graphically. (<u>very</u> large graph)
2. Elicit advantages and disadvantages of solution by graphing:
   <u>Advantages</u>:
   a) The method of graphing gives a picture (like a photograph) of the solution set. We can determine the exact number of solutions at a glance and know that we have all possible solutions.
   b) The method of graphing is the <u>only</u> method for finding the solution set of the system of inequalities.

Disadavantages:

a) Graph paper is needed.
b) Graphing will not give answers which are exact if the solution set has very large numbers, very small numbers, or fractional values.
c) Graphing is time consuming.

3. Have students use the addition property of equality to add the equations of the Motivation: $4x + y = 7$; $2x - y = 8$. The new equation is: $6x = 15$ (only one variable). Find the value of x. We now return to either one of the original equations to find the value of y. Note that solution set is identical to that found by graphing; check exactly as before.

APPLICATIONS:

Use the method of elimination by addition to solve the following systems of equations:

1. $x - y = 12$
   $2x + y = 3$

2. $-x + y = 1$
   $3x - y = 1$

2. $3x + 4y = 15$
   $-3x + y = 0$

4. $2y - 2x = -4$
   $2x + y = 4$

SUMMARY:

1. Today we learned to use the method of elimination by addition for a system of linear equations. How does this method work? (If x or y variables have the same coefficient but opposite signs, we can use the addition property of equations to eliminate that variable.)
2. What are the advantages of using the algebraic method of elimination by addition?
3. Why is it important to verify that the solution-pair satisfies both equations?

HOMEWORK:

1. Graph each pair of equations and their sum equation. Check that their graphs intersect in a common point

yielding their common solution. Check that the coordinates of this point do satisfy the equation in each case.

a) $3x - y = 2$
$x + y = 6$

b) $2x - 3y = 1$
$x + 3y = 5$

c) $4x + 3y = 5$
$-4x + y = -9$

d) $x - 4y = -6$
$3x + 4y = -2$

2. Solve each system of equations and check your solution (need not graph).

a) $-3x + y = 4$
$3x + 4y = 1$

b) $1.5x + 3y = -3$
$-1.5x + 2y = -5$

c) $x + 2.5y = 2$
$-2x - 2.5y = 1$

d) $1.5x + \frac{1}{3}y = -1$
$-1.5x + \frac{2}{3}y = 7$

*3. Graph the solution set for each of the following:

a) $(x \geq -1) \wedge (x < 4) \wedge (y = x + 1)$

b) $(x = 3) \vee (y = 4)$

c) $(x = 3) \vee (y > 4)$

d) $(y < 2x) \longrightarrow (x + 2y = 10)$

e) $(y = 2x) \longrightarrow (x + 2y = 10)$

4. Challenge Exercise:

Solve graphically and algebraically: $y = 2x - 5$
$y = 2x + 7$

Explain each result and explain how they confirm each other.

Lesson 59 - Coordinate Geometry (#12)

AIM: How do we solve more difficult systems of simultaneous linear equations by algebraic methods?

PERFORMANCE OBJECTIVES: The student will be able to...

1. Manipulate the coefficients of a system of two linear equations so that one of the variables may be eliminated by the addition method.
2. Confirm graphical solution by algebraic methods.
3. Check solutions to systems of linear equations.

VOCABULARY: constant term, equivalent equation, elimination
review: muliplicative property of equations

MOTIVATION:

Consider the system of equations: $3x + y = 6$
$x + 3y = 10$

What happens when you add these equations?

DEVELOPMENT:

1. If we apply the method employed yesterday, we arrive at a new equation, $4x + 4y = 16$. This equation is <u>equivalent</u> to $x + y = 4$. However, we still have one equation with two variables. This has an infinite number of solutions, the same number of solutions as there are points on a line. How can we find the solution set that will satisfy all four of these equations?
2. Focus on the original system: $3x + y = 6$
$x + 3y = 10$
In the previous lesson, one of the variables "dropped out" when the equations were added. Elicit suggestions as to how that can be accomplished here. Review the multiplicative property of equality: "If both members of an equation are multiplied by a nonzero number, an

equivalent equation is formed".

Multiply both numbers of the first equations by -3, leaving the second equation alone: $-9x - 3y = -18$

$x + 3y = 10$

Proceed as with the previous lesson. (Find the solution set and check in the <u>original</u> equations.) Why check the <u>original</u> equations? (There may have been an error in manipulation to arrive at the derived equations.)

Show that each of the equations in the motivation problem are satisfied by the solution set, $\{(1,3)\}$.

Ask: "Could we have eliminated x rather than y?" (Yes, by multiplying the <u>second</u> equation of the original system by -3.)

3. Use $3x + 7y = -2$ / $2x + 3y = -3$ to demonstrate manipulation of both equations may sometimes be necessary.

APPLICATIONS:

Choose from:

1. $4x + 5y = 23$ / $4x - y = 5$
2. $5a - 2b = 3$ / $2a - b = 0$
3. $2x + 5y = 20$ / $x - 3y = -1$
4. $2x - y = 5$ / $2x - y = -7$
5. $7x = 5 - 2y$ / $3y - 16 - 2x$

(Show how this system confirms graphical methods.) (Inconsistent // lines, no solution.)

SUMMARY:

Elicit the procedure for solving a system of simultaneous linear equations by the addition method:

1. Transform each equation so that variables appear in one member and a constant is in the other member.
2. Use the multiplicative property of equality to have one of the variables with the same coefficient with opposite sign as in the other equation.
3. Eliminate one of the variables by the method of elimination by addition.
4. Find the solution set and check.

HOMEWORK:

1. Solve for both x and y using addition and the multiplicative properties of equations in each case. Check their common solution.

   a) $2x + y = -1$
      $x - 2y = -3$

   b) $3x + 2y = -7$
      $-x + 2y = -3$

   c) $x + 1.5y = 0$
      $x - 3y = -9$

   d) $3x - 2y = -10$
      $4x - y = -10$

2. Solve: $\frac{x}{2} + \frac{y}{3} = 3$, and check.
   $\frac{x}{3} + \frac{y}{2} = \frac{1}{3}$

3. Challenge Exercise:
   Use a system of linear equations to solve:
   Three shirts and two ties cost $53.00 while two shirts and three ties cost $47.00. What is the cost of one shirt and one tie?

Lesson 60 - Coordinate Geometry (#13)

AIM: How do we solve a system of linear equations algebraically by the method of substitution?

PERFORMANCE OBJECTIVES: The student will be able to...

1. Solve systems of linear equations in two variables by the method of substitution.
2. Check the solution by replacing variables in the original equations.
3. Apply algebraic methods of solution to verbal problems.

VOCABULARY: method of substitution
vocabulary review: absolute value, coefficient

MOTIVATION:

Solve and check the system of equations:

$$x = 3y + 1$$
$$2x - 5y = -1$$

DEVELOPMENT:

1. This system can be solved by the method of elimination by addition; but there is a more effective method when one of the equations is solved for one variable in terms of the other. Note that in the first of this pair of equations, we have x solved in terms of y.
   Since x and 3y + 1 represent the same number, we can replace x by 3y + 1 in the second equation. This substitution yields a second equation that contains only one variable, namely:
   $$\begin{aligned} 2(3y + 1) - 5y &= -1 \\ 6y + 2 - 5y &= -1 \\ y + 2 &= -1 \\ y &= -3 \end{aligned}$$
   (Emphasize that this is the substitution that gives the method the name.)
   Since we now know the <u>value</u> of y, we can find the value of x by replacing y by -3. Thus, the solution set is $\{(-8,-3)\}$.

For those who feel comfortable with this method of solving a system of linear equations in two variables, it works best when the absolute value of one of the coefficients is one.

2. Elicit the summary of the new method: To solve a system of linear equations in two variables by the method of substitution:
   1) Transform one of the equations into an equivalent equation in which one of the variables is expressed in terms of the other (if necessary).
   2) Eliminate one of the variables in the other equation by replacing it with the result of 1 above.
   3) Solve the resulting equation which now has only one variable.
   4) Use the result of 3 to find the value of the second variable.
   5) Check as in previous lessons.

APPLICATIONS:

Solve each system of linear equations by the method of substitution:

1. $2x + 3y = 11$
   $y = 1 - 2x$

2. $y = 2x + 14$
   $x - 2y = -13$

3. $2x - 9y = 3$
   $x - 3y = 6$

4. $2x - y = 2$
   $3x + y = 3$

Note: The use of the substitution method in systems 3 and 4 is somewhat awkward. First, one equation must be solved for one of the variables.

5. A package of 5 shaving blades costs 35¢. A package of 25 shaving blades costs $1.55. Assuming that the cost for packaging for both 5 and 25 blades is the same, find the actual price for a blade and the packaging.

6. The perimeter of a rectangle is 60 in. If the length is doubled while the width is tripled, the new perimeter becomes 140 in. What are the dimensions of the original rectangle?

SUMMARY:

1. Name the three methods we have learned for finding the solution set of a system of simultaneous linear equations.

2. Tell which method you would select for each of the following systems. Explain your choices.

   $y = 5x - 3$ $\quad 2x + 3y = 4$ $\quad 2x - 3y = 8$

   $y = -2x + 11$ $\quad -3x + y = -17$ $\quad y = 2x + 4$

   What other factors would you consider in selecting a method?

HOMEWORK:

1. Solve using the method of substitution. Then check your solution in both equations.

   a) $3a + 2b = 20$
      $b = 2a - 11$

   b) $r = 3s + 5$
      $2r + 3s = -1$

   c) $6u + v = 20$
      $4u + 5v = 22$

2. Solve two ways by any methods. Check your solution in both equations.

   a) $5x + 2y = -3$
      $x - y = -2$

   b) $2x = 4y + 1$
      $4x - 2y = 8$

   c) $1.5x + y = 1$
      $2.5x + 3y = 11$

3. The smaller of two numbers is 6 less than twice the greater. If their difference is 1, find the two numbers.

4. Graph: $\left[(y > x - 1) \wedge (y < 5)\right] \vee (y = x)$.

5. <u>Challenge Exercise</u>:

   A father is 3 times as old as his son. In twelve years, the father will be twice as old as his son. What are their present ages? Let x represent the father's age and y represent the son's.

   $x = 3y$

   twelve years hence: $x + 12 = 2(y + 12)$

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